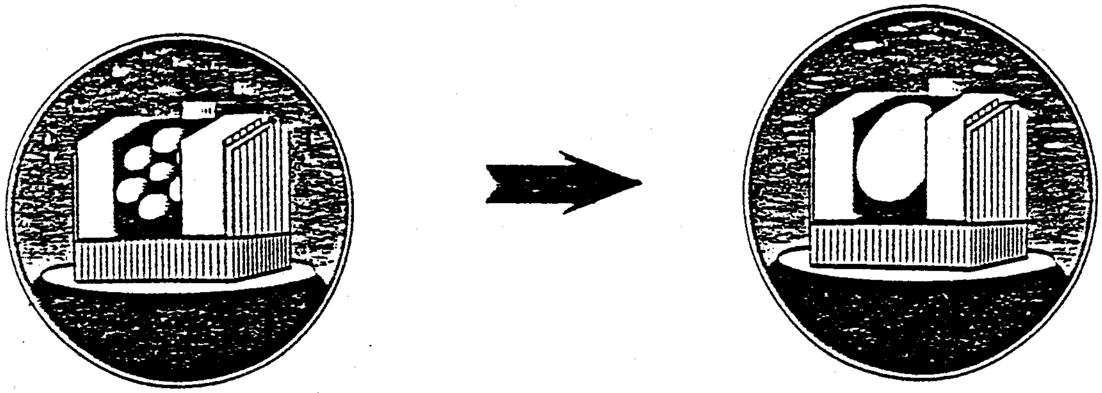
6.5 METER TELESCOPE



MMT Conversion Technical Memorandum #92-3

Fatigue Limits of a 2.2m Stressed Lap Baseplate
on the MMT and Columbus Mirrors

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1.0 Abstract

This report briefly investigates the fatigue strengths of aluminum alloys relative to the expected alternating stress encountered during polishing, and estimates the statistical likelihood of baseplate failure. The stresses are calculated using the GIFTS finite element program. The stress model is independent of the method used to bend the plate because the deflections are input directly onto the nodes of the model. Both the central and azimuthal variation of the peripheral stresses are evaluated. A subsequent refinement of the model will be required in order to include localized stresses due to the actuators or the effects of weight relieving the plate. Finally we will recommend which alloys will be appropriate for a 2m+ baseplate and what conditions should be followed during manufacturing. The results show that a 7075-T6 baseplate is expected to survive at least 5-10 mirrors.

2.0 Lap Geometry

The calculations are for a 2.2m x 4i thick lap baseplate moving on the Columbus 8.4m f/1.14 and MMT 6.5m f/1.25 mirrors. A 2.2m diameter lap was chosen because it is the largest piece of 7075-T6 x 4i aluminum that can be made at the foundries (a 2.5m 4i or 6i thick piece of 6061-T6 can be purchased, but the fatigue strength of 6061 is half that of 7075).

Plate deflections are calculated with a modified version of B. Martin's zdcalc() which is altered to include (not subtract) the preload deflections[1]. In this way, the absolute plate stresses are calculated in GIFTS directly. The preload radius is estimated to be 120m giving 5.4mm of edge sag. This is twice the expected deflection of the lap from the reference shape for either mirror. In order to minimize the plate stress, one would like to adjust

1. The preload is the curve of the lap when it is centered over the vertex of the mirror. All other lap shapes are defined relative to this reference. The reference shape has power because at all other locations on the mirror, the lap has less curvature.

the preload sag to a value only slightly greater than the expected deflection, but experience optimizing the 1.2m Phillips Lab and 0.6m Lennon stressed laps has shown that a larger preload is required.

3.0 Properties of Aluminum Alloys and Availability

Table 1 lists the strength properties of Al alloys and clearly illustrates that wrought Al is much stronger than cast. Alloys are listed in descending order of ultimate strength.

TABLE 1. Strength Properties of Various Al Alloys.

Alloy/Temper	Ultimate Strength kpsi	Yield Strength kpsi	Fatigue Strength kpsi @ 50Mcycles
7075-T6(wrought)	82	72	24
2014-T6(wrought)	70	60	18
2024-T4(wrought)	68	48	20
2219-T87(wrought)	61	49	?
6061-T6(wrought)	45	40	13.5
Typical (cast)	35	25	9
K-100 (cast)	33	22	?

Large plates of Al may be obtained from both Reynolds Al Co. and the Aluminum Corp. of America (ALCOA). Table 2 summarizes recent bids. Delivery is 3-4 weeks for K-100 and up to 12-14 weeks for 7075. Any piece of 7075 larger than 2.14m is delivered with the "O" temper, and must be naturally aged (for years) to achieve the strengths listed in table 1. The prices do not include any kind of machining. The wrought alloys must be cut (and purchased) from larger rectangular ingots. The density of Al is $2.7 Mg/m^3$.

TABLE 2. Recent Bids for Large Al Plates.

Alloy/Temper	Size	Weight (lbs)	Min mill order price
K-100	2.5m x 4-6i	2900(4i) 4500(6i)	Made to order ($16K/19K)
6061-T651	2.5m x 4-6i	"	8000 lbs, $18K @ 2.24/lb
7075-T651	2.14m x 4i	2170	8000 lbs, $25K @ 3.14/lb

4.0 Fluctuating Stress Fatigue

Fatigue is considered to be the gradual deterioration of a material that is subjected to repeated loading. The fatigue limit of the baseplate depends on the number of stress cycles N, the mean stress (σ_m), and the amplitude of stress variation (σ_a). A fatigue diagram defines the endurance envelope of the material as a function of these parameters. The line connecting the fatigue (endurance for steels) and ultimate tensile stresses is called the Goodman line and is considered to give the most conservative estimate to cyclic fatigue failure.

The fatigue strength (S_f) depends upon the number of stress reversals. The ultimate strength (S_{ut}) of a material is the fatigue strength for N=1/2. A plot of N vs. fatigue strength is called an SN diagram. To develop and SN diagram for Al, we use the formalism developed for steel and substitute the fatigue strength of Al @ 50Mcycles (S_{50}) for the endurance strength of steel:

$$S_f = aN^b$$

where

$$\left(a = \frac{(0.9S_{ut})^2}{S_{50}}\right); \left(b = -\frac{1}{3}\log\frac{0.9S_{ut}}{S_{50}}\right)$$

In actuality, many factors modify S_f:

$$S_f' \approx S_f \kappa_a \kappa_b \ldots \kappa_e$$

- The surface factor κ_a ranges from 1 for highly polished, 0.9 for a ground surface, to 0.4 "as rolled" for 7075. Clearly we will require the tension surface of the lap to be ground or polished.
- The size factor κ_b expresses the fact that the larger the volume subjected to repeated bending or torsion, the larger the probability of failure. For a 2.2m diameter baseplate, the factor is approximately 0.54.
- The load factor κ_c expresses a failure modification due to the type of loading. We take the average between bending (1) and torsion and shear (0.58) to get 0.8.
- The temperature factor κ_d is 1 for room temperature.

- Two components of the miscellaneous-effects factor κ_e are important for this work--reliability and stress concentration. Because materials manufactured with the same process actually have a distribution of strengths, reliability reduces the fatigue strength in order to express statistical confidence that the part will meet specification. A reliability factor of 1 represents a 50% chance of the part meeting the specification. We choose 0.70 or a 99.99% certainty. Stress concentration results whenever some feature perforates a surface in tension. Here, holes from two sources must be considered. The first are actuator attachment holes put along the side of the plate. It is difficult to estimate the effect because the actuator bracket is also held by friction due to the bolt tension, and a shoulder bolt scheme is used to reduce tension on the threads. We estimate 0.75. The second are holes in the tension work surface of the plate such as those to attach the kinematic tooling balls. Here a notch radius of 4mm is required to achieve 0.8. However, we suggest gluing perforated pucks onto the baseplate to achieve 1.

So we have: $S_f' \sim 0.9 \cdot 0.54 \cdot 0.8 \cdot 1 \cdot 0.52 \rightarrow 0.20 \cdot S_f$ (or $0.16 \cdot S_f$ if the work surface is very carefully perforated for the kinematic tooling balls).

Figure 1 shows the resultant SN diagrams for 7075-T6 and 6061-T6 Al which we consider to be the most readily available of the wrought alloys in large sizes. Solid lines show the respective fatigue strengths and dotted show the modified strengths without kinematic support holes.

5.0 GIFTS Modelling

5.1 Geometry

Figure 2 shows the geometry of the baseplate model used for the calculation of plate stresses. The plate deflections (including preload) are input to nodes of the shell elements. The azimuthal peripheral stresses are sampled on the inner annulus of quadrilateral elements. The exterior quadrilateral elements displace the edge effects of the model away from the sampled elements.

5.2 Results

The peripheral stresses of the plate are shown in Figure 3. The azimuth angle is 180 degrees towards the mirror vertex (0 away). The upper curve represents the azimuthal stresses for the preload. The lower two curves show the peripheral plate stresses at the edge of each mirror (R denotes the distance of the lap center from the vertex). The stresses are virtually identical and illustrate that, compared to the Columbus mirror, the slower 6.5m

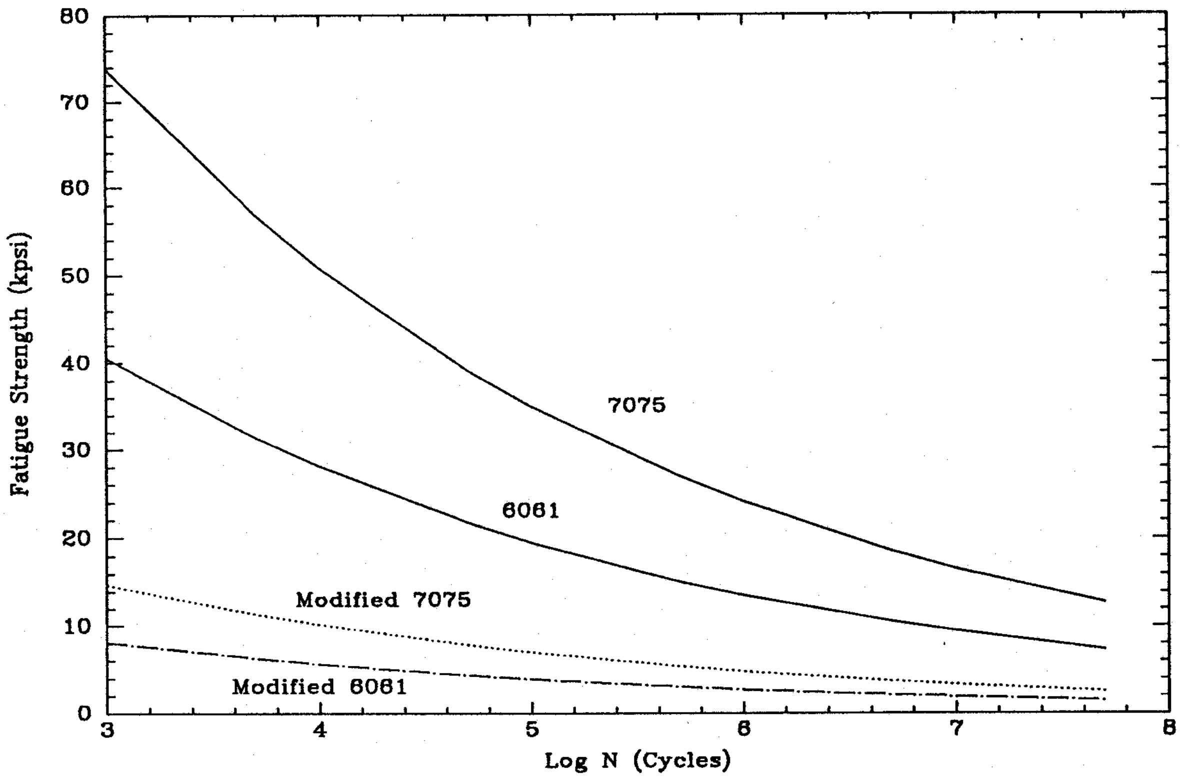

Figure 1 SN diagrams for 7075 and 6061 alloys. The solid curves show fatigue strength, and the dashed curves show fatigue strength modified appropriately for the baseplates (multiply the latter by 0.8 to include carefully contoured kinematic support holes drilled into the tension side).

mirror compensates the fact that the lap is a larger fraction of its diameter.

In addition, the stress at the center of the plate shows a variation ranging from 6630psi at the vertex to 4940 and 5110psi for the Columbus and MMT mirrors respectively.

6.0 Discussion

6.1 Analytic Check

We perform an analytic check of the preload stress as follows. The moment applied to the plate is the stiffness D multiplied by the sum of the curvature in its own direction and Poisson's ratio times the curvature in the orthogonal direction (*e.g.* Hartog 1952). The bending moments in the radial and tangential directions are:

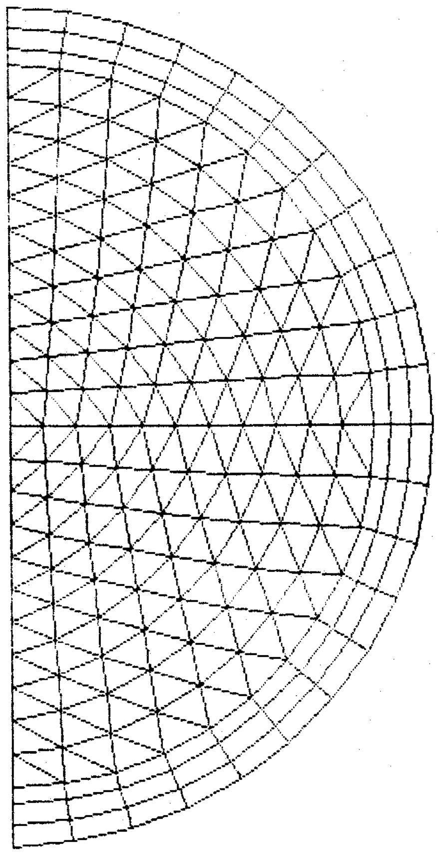

Figure 2 Geometry of the GIFTS finite element model used to calculate the baseplate stresses.

$$M_r = D\left(\frac{\partial^2 w}{\partial r^2} + \frac{\nu}{r}\frac{\partial w}{\partial r}\right); M_t = D\left(\frac{1}{r}\frac{\partial w}{\partial r} + \nu\frac{\partial^2 w}{\partial r^2}\right)$$

$$D = \frac{Et^3}{12\,(1-\nu^2)}$$

"w" is the plate deflection, C=1/2R is the plate curvature, t is the thickness, E is Young's modulus (70.3×10^9 Pa for Al), and ν is Poisson's ratio (0.345 for Al). For spherical bending, $w=Cr^2$,

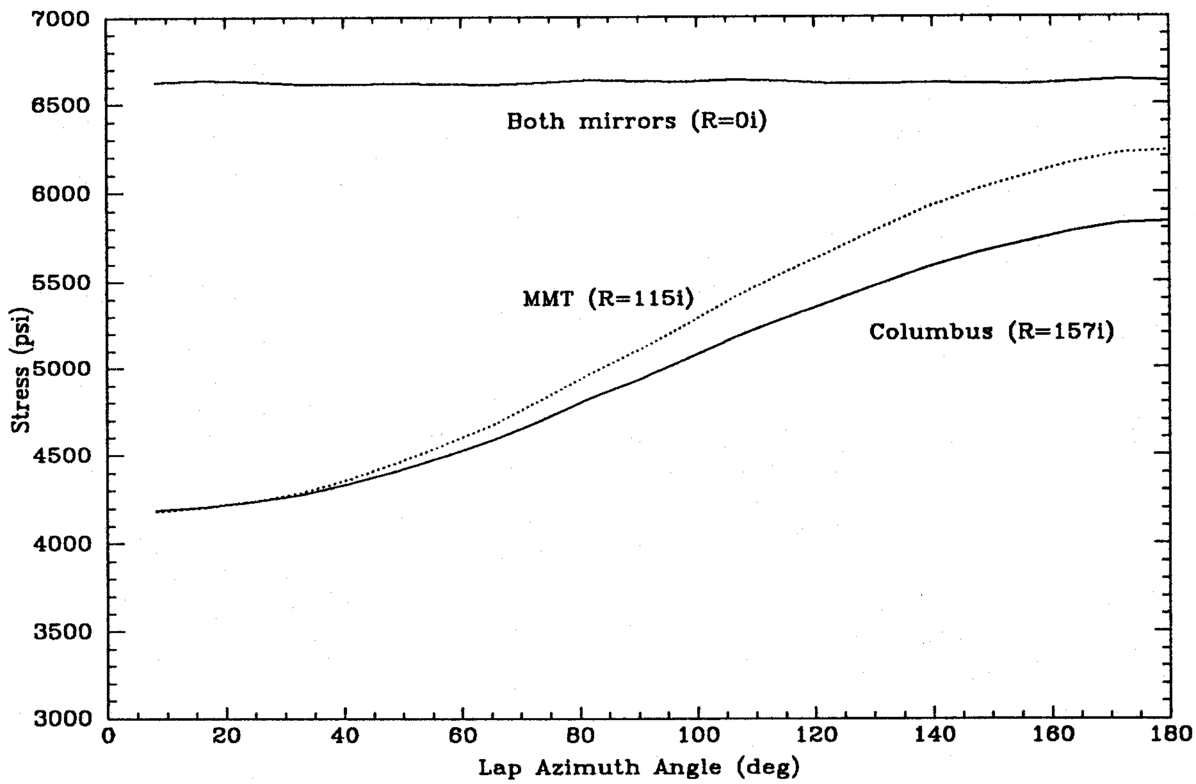

Figure 3 The peripheral stresses of the 2.2m stressed lap plate on the Columbus and MMT mirrors. "R" denotes the distance of the lap from the mirror vertex. See text for details.

$$M_r = 2CD(1+\nu) = M_t$$

The maximum stress is:

$$S_{max} = \frac{6M_r}{t^2} = \frac{CEt}{1-\nu} \rightarrow 6600psi$$

It's interesting to note that the highest stress is always produced by the preload since at any other location on the mirror, the curvature is smaller (even though the forces on some actuators are larger).

6.2 Goodman Relations

Two simplified types of alternating stresses will be considered here: 1) the cycling that occurs for a point on the periphery of the lap as the lap rotates near the edge of the mirror, and 2) stress cycling that occurs for the center of the baseplate as the lap is moved from mirror vertex to edge with the horizontal ways. The longevity of the plate center is significantly longer than the edge because the cyclic frequency is much lower (typ. 0.5 rpm compared to 5-10 rpm).

Modified Goodman relations (at $N=10^6$ and $N=10^7$) are shown in Figure 4 for both

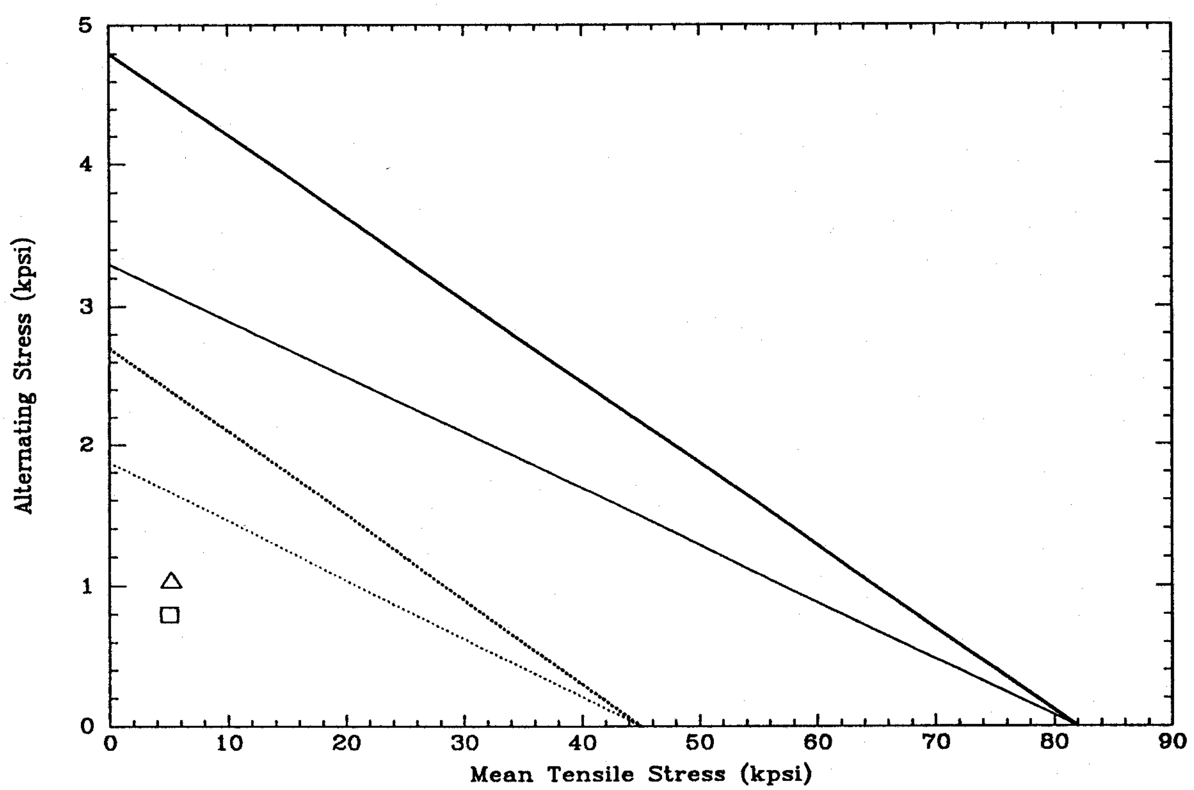

Figure 4 Modified Goodman relations for 7075 (solid lines) and 6061 (dashed lines) alloys calculated from section 4.0 for logN = 6 (heavy lines) and logN = 7 (light lines). The alternating and mean stresses of the 2.2m baseplate are shown for the center (triangle) and periphery (square).

7075 and 6061 alloys. The safety factors for both the center and edge at $N=10^6$ are approximately 6 and 3.4 for 7075 and 6061, and narrows to 4 and 2.4 for $N=10^7$. The safety (or confidence) factor (S_f'/σ_a) signifies how much greater the fatigue strength is than the stress amplitude. These Goodman relations do not include the effect of any perforation in the tension side of the plate aside from surface finish.

6.3 Polishing Cycles

The lifetime of the plate can be estimated by first estimating N for polishing one mirror. We assume that the lap, on average, rotates at 5 rpm and is completely translated on the mirror from vertex to edge at 0.5 rpm. Table 3 summarizes the expected lifetime of the plate center under translation and the plate edge under rotation. The stress amplitude and mean are shown as well as the safety factors for the alloys. The lifetimes are expressed in terms of the number of mirrors that can be polished in $N=10^6$ cycles by assuming that completing one mirror requires 8 months @ 2 hours/day for 20 days/month (including calibration and periodic testing). Using the average speeds noted above, we obtain $N=10^5$ rotations and $N=10^4$ translations per mirror.

TABLE 3. Estimated plate survival for $N=10^6$ for a 2.2m lap on a Columbus mirror.

Stroke	$\sigma_a kpsi$	$\sigma_m kpsi$	Safety(7075)	Safety(6061)	#mirrors to $N=10^6$
Translation	1.1	5.9	4.3	2.4	100
Rotation	0.8	5.2	6.0	3.4	10

Clearly, rotation fatigues the plate at the greatest rate. <u>We speculate that localized actuator stresses may reduce the lifetime by 2, however the baseplate can survive 10^7 cycles if necessary with a generous 7075 safety factor.</u> Any reductions in the preload curvature will increase the safety factor dramatically. These results are encouraging for 6061, and it looks as if a 2.5m baseplate could also survive.

6.4 Future Work

A highly refined finite element model that includes the actuators must be developed. This will enable us to investigate localized actuator stresses as well as better predicting the bending forces required. Such a model must include a realistic approximation of the edge stiffening due to the actuator attachments, as well as stress concentrations at the actuator attachments.

6.5 Manufacturing Considerations

- The tension surface should be ground or polished.
- We recommend that no holes be drilled into the tension surface and that the kinematic support holes be put into "pucks" which are then glued onto the surface.
- If necessary, any notch in the tension surface should have a 4mm or larger fillet, and the effect will reduce the fatigue strength by 20%.
- Considerable forces will be applied the actuators. Ideally, we recommend that the actuator mounting method not include tapped and drilled holes, but rather some sort of "clip-on" friction fitting device. The shoulder bolt attachments used in the previous laps should work well, and should be attached near the neutral plane to avoid stress concentration from the plate bending.

6.6 Acknowledgments

The above analysis strictly follows the formalism set forth in Mechanical Engineering Design, fifth edition by J. Shigley and C. Mischke (chapter 7, Variable Loading).

We thank Buddy Martin for a critical reading of this report and for pointing out several errors.