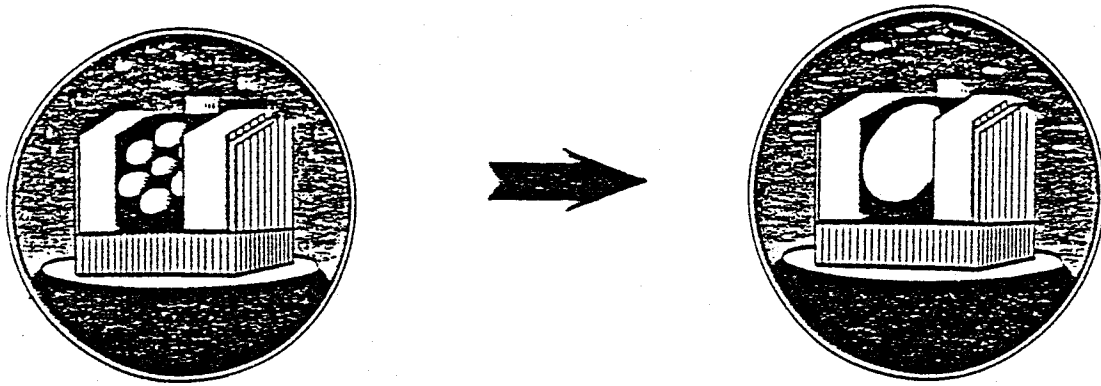


## 6.5 METER TELESCOPE



### **MMT Conversion Technical Memorandum #96-1**

A Hardpoint Length Calculator for the MMT Conversion

S. C. West

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# A Hardpoint Length Calculator for the MMT Conversion

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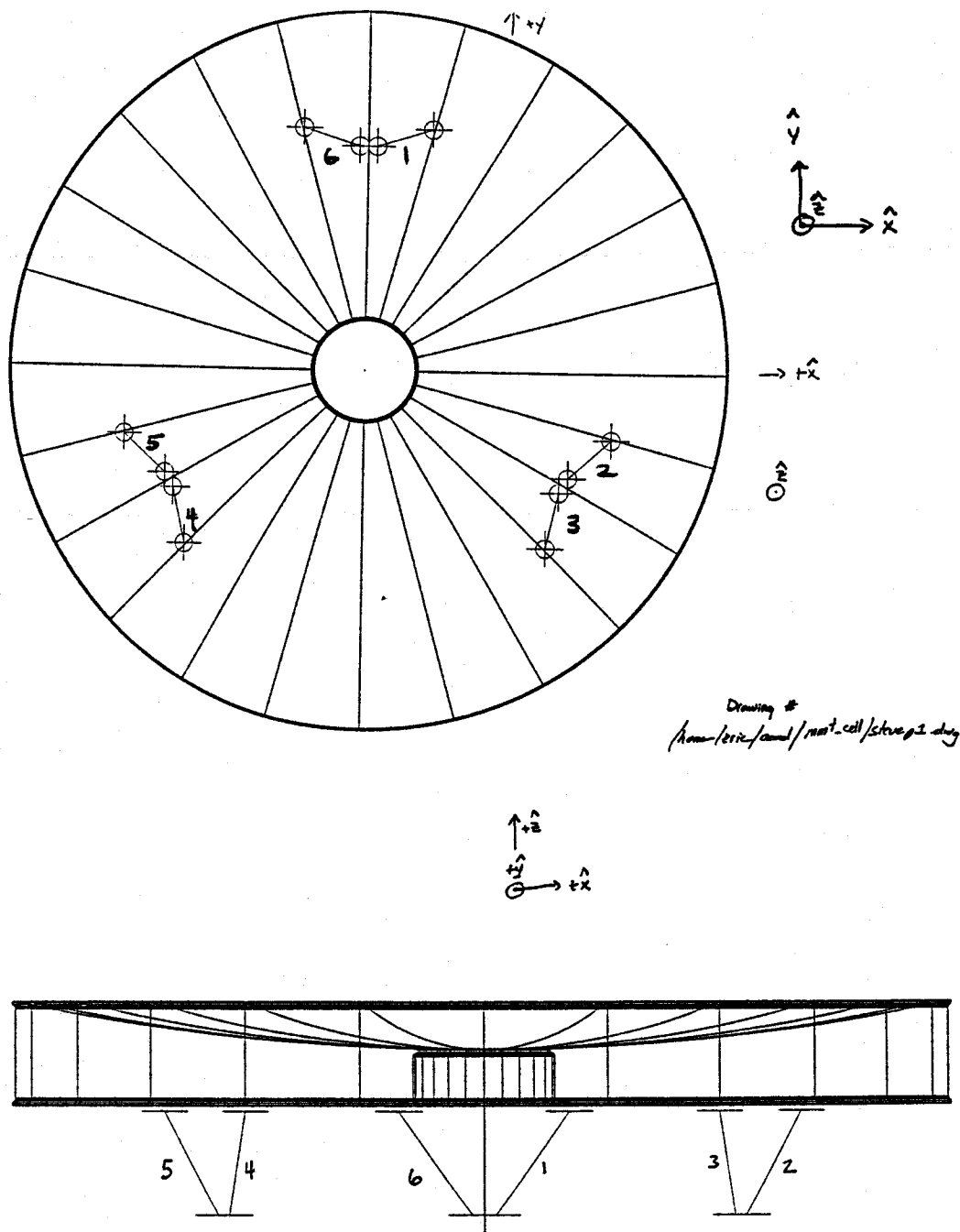
## 1.0 Overview

This memo provides positioning information for the 6.5-m primary mirror hardpoint system. In addition to providing the stiffness of the support system, the hardpoints allow one to precisely position of the primary mirror relative to its cell. A matrix that describes the changes in lengths of each hardpoint actuator as a function of mirror motion about the vertex is developed.

In the mechanical journals, the process of calculating the lengths of adjustable struts as a function of output plane (mirror) position in a parallel manipulator is termed the inverse kinematic solution. This is our task. For those interested, a literature summary of these mechanisms is cited [1-6], and much effort has been spent determining generalized analytic solutions for the direct kinematic problem of determining the position and orientation of the output plane given arbitrary lengths of the strut actuators (a much harder problem).

## 2.0 Geometry

The 6.5-m primary mirror hardpoint system can be described as two planes (the cell backplate and the mirror backplate) whose orientation with respect to each other is controlled with 6 adjustable struts (hardpoints) containing flexures near each attachment. The hardpoints are grouped into 3 pairs with a near-common connection at the cell backplate and separate connections onto the mirror backplate. Such a configuration is commonly called a 3-6 Stewart platform parallel manipulator (*i.e.*, the struts are connected at 3 locations on the fixed plane and 6 locations on the output plane). The exact geometry for the MMT 6.5-m mirror cell is shown in *Figure 1 on page 2*. The coordinates of the attachment points (as of 2-21-96) are:

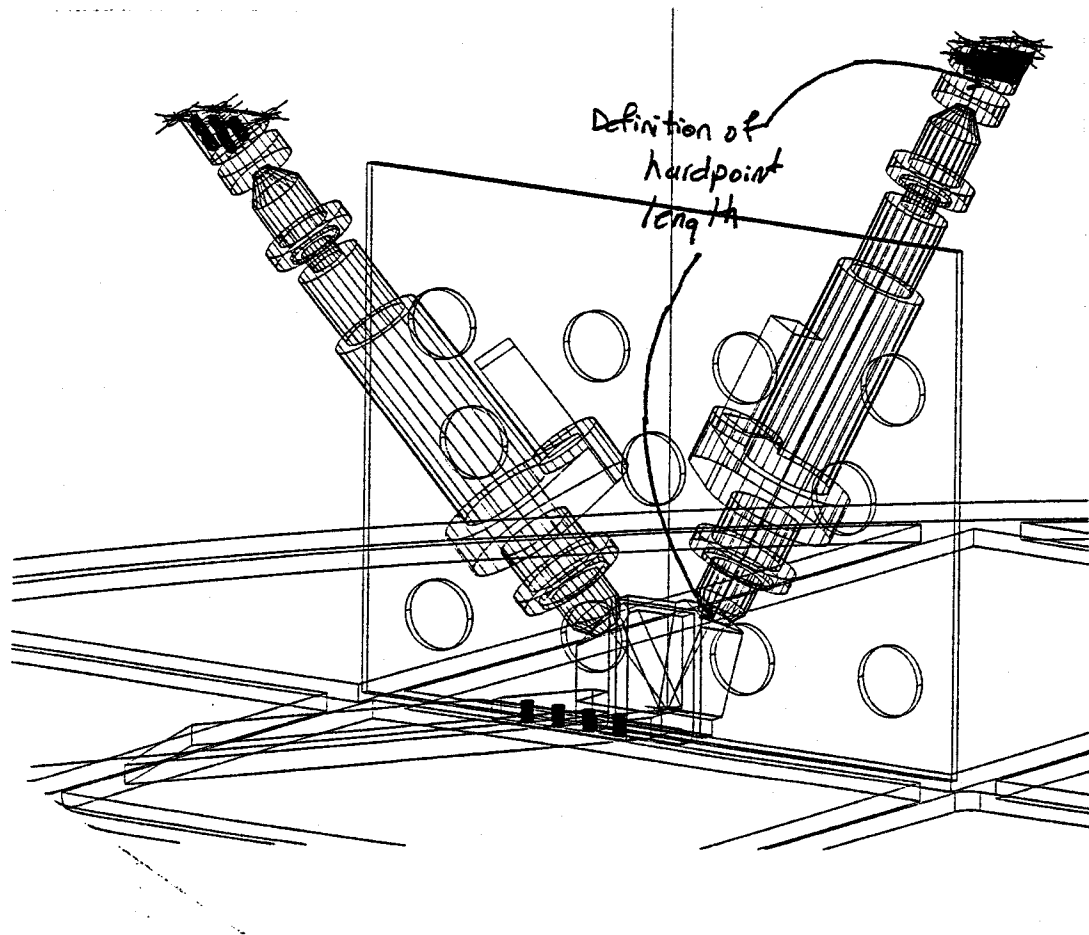


**FIGURE 1. Top and side wireframe views of the MMT hardpoint attachments. The official numbering scheme is shown in both views. The upper hardpoint attachment plane (through the E6 wedge faces) is separated from the lower attachment plane by 715.12 mm. Drawings by Eric Anderson.**

**TABLE 1. Hardpoint attachment coordinates from E. Anderson's ACAD 3-D layout. The mirror vertex (in the operating position with the support system on) is defined to be 0,0,0. Model construction is responsible for the small errors in symmetry.**

Hardpoint #	Lower Plane (mm)			Upper Plane (mm)		
	x	y	z	x	y	z
1	78.125	2042.798	-1136.274	573.334	2198.675	-421.152
2	1808.178	-953.742	-1136.274	2190.77	-602.828	-421.152
3	1730.052	-1089.055	-1136.274	1617.442	-1595.859	-421.152
4	-1730.052	-1089.055	-1136.274	-1617.449	-1595.849	-421.152
5	-1808.178	-953.742	-1136.274	-2190.775	-602.816	-421.152
6	-78.125	2042.798	-1136.274	-573.321	2198.677	-421.152

The upper attachments are defined at the centers of the faces of the E6 wedges where the hardpoints attach. The lower connections are at the centers of the faces of the lower cell connection blocks (Figure 2 on page 3).



**FIGURE 2. Upper and lower connection point definitions. Drawing by Eric Anderson.**

### 3.0 Hardpoint length changes vs. mirror motion

Simple translations and rotations of the mirror have a complex relationship to changes in the hardpoint lengths. In order to calculate these changes in hardpoint lengths, we consider the lower attachment plane to be fixed and the upper attachment plane to be moveable. The motion of the upper attachment points is given by applying rotations and translations about the mirror vertex. Specifically, the coordinate of an upper attachment is given by:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [E(\Delta\phi, \Delta\theta, \Delta\Psi)] \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

where  $x, y, z$  are the starting coordinates of the attachment point,  $\Delta X, \Delta Y, \Delta Z$  are mirror translations, and  $[E(\Delta\phi, \Delta\theta, \Delta\Psi)]$  is the Euler rotation matrix [7] given by:

$$[E(\Delta\phi, \Delta\theta, \Delta\Psi)] = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ \sin\psi\sin\theta\cos\phi - \cos\psi\sin\theta & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\theta & \cos\theta\sin\psi \\ \cos\psi\sin\theta\cos\phi + \sin\psi\sin\theta & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\theta & \cos\theta\cos\psi \end{bmatrix}$$

where  $\phi$  is a rotation about the  $z$  axis (yaw),  $\theta$  is about the intermediate  $y$  (pitch), and  $\psi$  is about the final  $x'$  axis (bank or roll). Positive angles are given by the right-hand rule.

Using the above equations, we can model the influence of each motion on the lengths of the hardpoints and obtain:

$$\begin{bmatrix} \Delta l_1 \\ \Delta l_2 \\ \Delta l_3 \\ \Delta l_4 \\ \Delta l_5 \\ \Delta l_6 \end{bmatrix} = \begin{bmatrix} -560.4 & -176.4 & -809.3 & 5.5 & 3.4 & -9.0 \\ -433.0 & -397.1 & -809.3 & -5.5 & 9.5 & 1.6 \\ 127.4 & 573.5 & -809.3 & 5.5 & 6.1 & 7.4 \\ -127.5 & 573.5 & -809.3 & -5.5 & -6.1 & 7.4 \\ 432.9 & -397.2 & -809.3 & 5.5 & -9.5 & 1.6 \\ 560.3 & -176.5 & -809.3 & -5.5 & -3.4 & -9.0 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix}$$

$\Delta l_i$  is the length change of the  $i$ th hardpoint ( $\mu m$ ),  $\Delta X, \Delta Y, \Delta Z$  are mirror translations (mm), and  $\Delta\phi, \Delta\theta, \Delta\psi$  are angles in arcseconds. The first 3 columns of the influence matrix have units of  $\mu m/mm$  and the last 3 columns have units of  $\mu m/arcsecond$ . The matrix elements remain linear over the allowed motions of the primary mirror. This matrix acts by performing rotations first, then translations. However, over the small operating volume of the mirror, these operations are very nearly commutative.

The absolute length of hardpoint i is:

$$l_i = l_{0i} - \Delta l_i$$

where  $l_{0i}$  is the nominal length between the hardpoint connection points for the mirror in the operation position with zero offsets or tilts. From Table 1 on page 3, this length is 883.70 mm.

We can immediately obtain the length of the hardpoints when resting on the static supports. Neglecting metrology errors, the static supports are 4-mm below the operating position of the mirror and the loaded axial deflection of each support is 3-mm (resulting from 19,000 lbs of mirror and attached hardware resting on 200 supports each with an axial compliance of 800-lbs/in). Therefore the hardpoint lengths at zenith-pointing with the support system turned off are approximately 878.04-mm.

**NOTE:** The lengths calculated in this memo are simple distances between the hardpoint attachment locations. In actuality, only the parts of the hardpoint that lie between the flexures remain colinear. The parts between the flexures and corresponding attachment points have an additional cosine term not considered here. Table 2 on page 5 gives the angles that

**TABLE 2. Hardpoint angles (degrees) with the cartesian axes.**

hp #	Nominal position			Lowered onto static supports			dx = 1mm from nominal		
	x	y	z	x	y	z	x	y	z
1	124.08	100.16	144.02	124.33	100.23	143.75	124.14	100.15	143.97
2	115.66	113.40	144.02	115.83	113.56	143.75	115.71	113.38	143.98
3	82.68	55.01	144.02	82.63	54.75	143.75	82.74	55.00	144.03
4	97.32	55.01	144.02	97.37	54.75	143.75	97.39	55.01	144.01
5	64.34	113.40	144.02	64.17	113.56	143.75	64.40	113.41	144.06
6	55.92	100.16	144.02	55.67	100.23	143.75	55.97	100.17	144.07

the hardpoints make with the x,y, and z axes for several mirror positions. From this, we find that for even the most dramatic excursions from the nominal position (e.g. mirror resting on the static supports), the hardpoints tilt at most 1 degree. Since the distance from the upper flexure to the upper attachment point is about 180mm, the maximum error of the hardpoint length is roughly 30 microns plus the effect calculated at the lower flexure. Over the normal operation volume of the mirror, the tilt effect is a small fraction of a degree, and the length error appears to be insignificant as illustrated in the above table by decentering the mirror 1-mm.

Thanks to Eric Anderson for verifying the accuracy of the equations of motion for the upper plane with a 3-D ACAD model. For those interested in the details, all calculations can be found in the MathCad document `hplength.mcd`.

## 4.0 References

1. D. Stewart 1965, "A Platform with Six Degrees of Freedom", *Proc. Instr. Mech. Engr.* **180**, 371-386.
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