Hardpoint platform matrices for the MMT 6.5-m primary mirror cell

S.C. West

1.0 Overview

This memo summarizes the conversion matrices related to the MMT primary mirror hardpoint platform. There are two matrices (and their inverses) of importance:

The first converts the six hardpoint force readings into the xyz forces and torques acting on the mirror, and is essential for the air-support system control loop (e.g. see Hill, “Mirror support system for large honeycomb mirrors”, UA-95-02 section 8 for the USAF values).

The other expresses the change in length of the hardpoints required to reposition the mirror in translation and/or rotations (e.g. West, “Hardpoint length calculator for the MMT conversion”, MMTC TM #96-1).

This memo provides the generalized vector equations for deriving these matrices and shows their forms for the MMT hardpoint platform. They should be helpful for hardpoint installation, for positioning the mirror in the cell, for the air-support control loop, and for using the platform as a force-torque sensor for sensing external mirror loads.

These matrices have now been cross-checked 3 ways: 1) using the generalized vector and Euler equations, 2) via E. Anderson’s 3-D ACAD model for positioning, and 3) via B. Cuerden’s ANSYS model for forces and torques.

2.0 Coordinate system

The coordinate system is defined with the origin at the mirror vertex. +Z points from the vertex upwards to the secondary mirror, +Y points straight up when the mirror is horizon pointing, and +X points rightward when the mirror is horizon pointing (to make a right-handed coordinate system). T_x, T_y, T_z are the torques applied around the X, Y, and Z-axes respectively (right-handed rotation about each axis is +).

The coordinates of the hardpoint “work-points” have been defined by Eric Anderson. The upper coordinates intersect the midplane of the mirror backplate, and the lower coordinates are the intersections of hardpoints pairs near the cell backplate. The origin of the coordinate system is the vertex of the mirror in its nominal operating position with the support system turned on. The hardpoint numbering scheme conforms to the project convention (i.e. hp#1 is located at roughly 12:30, and successive hardpoints are numbered clockwise, so that the pairs are 1 and 6, 2 and 3, and 4 and 5). The workpoint coordinates are:

TABLE 1. Upper workpoints

HP #	X in	Y in	Z in
1)	23.89	86.98	-14.68
2)	87.27	-22.80	-14.68
3)	63.38	-64.18	-14.68
4)	-63.38	-64.18	-14.68
5)	-87.27	-22.80	-14.68
6)	-23.89	86.98	-14.68

TABLE 2. Lower workpoints

HP#	X in	Y in	Z in
1 & 6	0.00	79.46	-49.18
2 & 3	68.81	-39.73	-49.18
4 & 5	-68.81	-39.73	-49.18

Six hardpoint vectors $\overrightarrow{hp}_{1\ldots6}$ are defined with their bases at the upper workpoint and their lengths equal to the distance from the upper to lower workpoints (1082.9 mm).

3.0 Force-torque matrix

The six forces observed on the hardpoints ($f_{1\ldots6}$) can be converted to the solid-body forces and torques acting on the mirror ($F_{x,y,z}$, $T_{x,y,z}$). The solid body forces are simply given by the projections (direction cosines) of the hardpoint vectors onto each axis which are given by vector dot products:

$$F_x = \sum_{i=1}^{6} f_i \frac{(\overrightarrow{hp}_i \cdot \hat{x})}{|\overrightarrow{hp}_i|} = \sum_{i=1}^{6} f_i (\hat{hp}_i \cdot \hat{x})$$

where $\hat{x}$ is the unit vector along the x-axis. Similar equations follow for the y and z-axes.

The solid-body torque about the x-axis is given by:

$$T_x = \sum_{i=1}^{6} f_i (\hat{x} \cdot (\vec{r}_i \times \hat{hp}_i))$$

where $\vec{r}_i$ are the vectors from the origin (vertex) to the upper workpoints, and similar equations follow for the other axes. For our axisymmetric platform, $\left|\vec{r}_i\right| = 2321.2mm$.

So for the MMT hardpoint geometry, we obtain:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} -0.560 & -0.433 & 0.127 & -0.127 & 0.433 & 0.560 \\ -0.176 & -0.397 & 0.574 & 0.574 & -0.397 & -0.176 \\ -0.809 & -0.809 & -0.809 & -0.809 & -0.809 & -0.809 \\ -1853.621 & 320.565 & 1533.045 & 1533.045 & 320.565 & -1853.621 \\ 699.999 & 1955.204 & 1255.265 & -1255.265 & -1955.204 & -699.999 \\ 1130.986 & -1130.992 & 1130.992 & -1130.992 & 1130.992 & -1130.992 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix}$$

where the forces are measured in N and the torques in Nmm.

The inverse of this matrix transforms the solid-body forces and torques into hardpoint forces:

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} = \begin{bmatrix} -0.675 & 0.255 & -0.206 & -2.041\times10^{-4} & -6.426\times10^{-5} & 1.474\times10^{-4} \\ -0.117 & -0.712 & -0.206 & 1.577\times10^{-4} & 1.446\times10^{-4} & -1.474\times10^{-4} \\ 0.558 & 0.457 & -0.206 & 4.641\times10^{-5} & 2.089\times10^{-4} & 1.474\times10^{-4} \\ -0.558 & 0.457 & -0.206 & 4.641\times10^{-5} & -2.089\times10^{-4} & -1.474\times10^{-4} \\ 0.117 & -0.712 & -0.206 & 1.577\times10^{-4} & -1.446\times10^{-4} & 1.474\times10^{-4} \\ 0.675 & 0.255 & -0.206 & -2.041\times10^{-4} & 6.426\times10^{-5} & -1.474\times10^{-4} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ T_x \\ T_y \\ T_z \end{bmatrix}$$

4.0 Translation-rotation matrix

So that we have both relevant matrices in the same memo, the matrix describing the changes in hardpoint lengths vs. the solid-body motion of the mirror (about its vertex) is:

$$\begin{bmatrix} \Delta l_1 \\ \Delta l_2 \\ \Delta l_3 \\ \Delta l_4 \\ \Delta l_5 \\ \Delta l_6 \end{bmatrix} = \begin{bmatrix} 560.4 & 176.4 & 809.3 & -5.5 & -3.4 & 9.0 \\ 433.0 & 397.1 & 809.3 & 5.5 & -9.5 & -1.6 \\ -127.4 & -573.5 & 809.3 & -5.5 & -6.1 & -7.4 \\ 127.5 & -573.5 & 809.3 & 5.5 & 6.1 & -7.4 \\ -432.9 & 397.2 & 809.3 & -5.5 & 9.5 & -1.6 \\ -560.3 & 176.5 & 809.3 & 5.5 & 3.4 & 9.0 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix}$$

where Δl_i is the length change of the ith hardpoint (μm), $\Delta X, \Delta Y, \Delta Z$ are mirror translations (mm), and $\Delta\phi, \Delta\theta, \Delta\psi$ are angles in arcseconds. ϕ is a rotation about the z axis (yaw), θ is about the intermediate y (pitch), and ψ is about the final x' axis (bank or roll). Positive angles are given by the right-hand rule.The first 3 columns of the influence matrix have units of $\mu m/\text{mm}$ and the last 3 columns have units of $\mu m/\text{arcsecond}$. Although this matrix was derived with the Euler rotation transformation, it can be seen that the first 3 columns are proportional to the direction cosines, and the last 3 columns are proportional to the torque vector equation given above (proportional terms given by units conversions and the distance of upper workpoint from the vertex).

Its inverse transforms the hardpoint length changes into the changes in the mirror orientation:

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{bmatrix} = \begin{bmatrix} 6.751\times10^{-4} & 1.168\times10^{-4} & -5.584\times10^{-4} & 5.584\times10^{-4} & -1.168\times10^{-4} & -6.751\times10^{-4} \\ -2.549\times10^{-4} & 7.121\times10^{-4} & -4.571\times10^{-4} & -4.571\times10^{-4} & 7.121\times10^{-4} & -2.549\times10^{-4} \\ 2.058\times10^{-4} & 2.061\times10^{-4} & 2.056\times10^{-4} & 2.063\times10^{-4} & 2.058\times10^{-4} & 2.062\times10^{-4} \\ -0.030 & 0.030 & -0.030 & 0.030 & -0.030 & 0.030 \\ 0.013 & -0.030 & -0.043 & 0.043 & 0.030 & -0.013 \\ 0.042 & -0.033 & -0.010 & -0.010 & -0.033 & 0.042 \end{bmatrix} \begin{bmatrix} \Delta l_1 \\ \Delta l_2 \\ \Delta l_3 \\ \Delta l_4 \\ \Delta l_5 \\ \Delta l_6 \end{bmatrix}$$