



MULTIPLE MIRROR TELESCOPE OBSERVATORY

Smithsonian Astrophysical Observatory and Steward Observatory, University of Arizona

M.M.T.O. INTERNAL TECHNICAL MEMORANDUM 88-1

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Subject: Software Limits on Telescope Performance: Considerations for
Catching and Tracking a Fast Satellite

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Software Limits on Acceleration and Velocity

	<u>AZ</u>	<u>EL</u>
Max Jerk	0.001073 deg/sec ³	0.001073 deg/sec ²
Max Acceleration	0.064373 deg/sec ²	0.064373 deg/sec ²
Max Accel achieved in	0.15 sec	0.15 sec
Max Velocity	1.33338 deg/sec	1.50 deg/sec
Max Velocity achieved in	20.8 sec	23.4 sec

(See later section for derivation of these figures.)

NOTE: All MAX VELOCITIES given are PEAK. They are reduced at high/low elevations. Acceleration and jerk maximums are constant, however.

For Satellite Tracking

For satellite tracking from a catalog of AZ/EL coordinates, at the instant of the starting catalog entry, even if the telescope is at the right position, the target will have some velocity and the telescope will be at rest. For fast satellites, the telescope may not be able to catch it until near the end of the trajectory. Thus, it is necessary to extrapolate the catalog back to some calculated telescope starting point in order to allow the telescope to achieve target velocity at the start of the real catalog. This study is the result of an unsuccessful attempt to track a satellite whose coordinates are listed in an appendix. Figures from this catalog are used in the following examples.

Time to achieve a velocity of V deg/sec is:

$$T = \frac{V}{A_{\text{Max}}} + \frac{1}{2} * \frac{A_{\text{Max}}}{J_{\text{Max}}}$$
$$= \underline{15.534 * V + 0.075} \text{ secs.}$$

For our example case, $V_{\text{AZ}} = 0.707$ deg/sec and $V_{\text{EL}} = 0.273$ deg/sec.

So, in this instance, AZ requires the most time to achieve initial velocity.

Using the above equation, $T = \underline{11.06 \text{ sec}}$

Therefore, extrapolating back about 15 sec should be safe.

To have target velocity achieved at the first of the real coordinates (match velocities at the FIRST coordinate, because you cannot start at the first coordinate and have expectations of matching it at the second), we have to extrapolate back two sets of coordinates. (Remember, satellite tracking starts at the second point in the catalog, not the first.)

Thus, make the two sets of coordinates spaced back by the extrapolation time you calculated, at a position assuming velocity constant at half the target velocity (this assumes constant acceleration to the target velocity, which is close enough).

Therefore, if you have target velocity V at the first satellite coordinate (time T_0), and have decided to extrapolate back by T seconds, then:

Coordinate 1	Time = $T_0 - 2*T$ Posn = $P_0 - 2*T * V/2$
Coordinate 2	Time = $T_0 - T$ Posn = $P_0 - T * V/2$
Coordinate 3	Time = T_0 (First actual satellite position.) Posn = P_0 (Vel = V)

In our example, $T = 15 \text{ sec.}$, $V_{AZ} = 0.707 \text{ deg/sec.}$, and $V_{EL} = 0.273 \text{ deg/sec.}$

Thus:

Coordinate 1	Time = $T_0 - 30 \text{ sec.}$ AZ = $-40.48047 - 10.605 = -51.0855 \text{ deg}$ EL = $32.36883 - 4.095 = 28.2738 \text{ deg}$
Coordinate 2	Time = $T_0 - 15 \text{ sec.}$ AZ = $-40.48047 - 5.303 = -45.7835 \text{ deg}$ EL = $+32.36883 - 2.047 = 30.3218 \text{ deg}$

and Coordinate 3 is the first coordinate of the original catalog.

Why We Couldn't Catch It

Without extrapolating back, we tried to catch and track the satellite using the catalog shown in the appendix. We actually caught up with it about 60 seconds into its 80 second pass. This is why:

We started at T0 with Telescope Velocity = 0 and Target Velocity = 0.707.

The acceleration phase of the telescope to max AZ velocity takes 21 seconds, by which time the target velocity was 1.05, and during which the following equations apply (approximately):

$$S_{tel} = (1/2) * a * t^2$$

$$S_{targ} = V * t \quad (V = \text{average velocity, near enough})$$

where S is angular distance travelled. Thus, at the end of 21 seconds

$$S_{tel} = 1/2 * 0.064 * 21 * 21 = 14.11 \text{ deg}$$

$$S_{targ} = (0.7 + 1.05)/2 * 21 = 18.38 \text{ deg}$$

Therefore, at the point where the telescope achieves maximum velocity, the target is leading the telescope by 4.27 deg.

At constant telescope max velocity of 1.333 deg/sec., and average target velocity over the next 20 - 40 seconds of approx $(1.05 + 1.22)/2 = 1.135$, the velocity differential is about 0.2 deg/sec.

The time to catch up 4.27 deg at 0.2 deg/sec = 21.35 sec

Therefore, total time to catch up = 42.35 sec

Note, this is the minimum time to catch the target, with no attempt to match velocities at the interception point, and assuming maximum achievable telescope velocity all the way. This is not the way the telescope control law works, of course. It will bring velocity error and position error to zero at the same point in time. Without trying to simulate the telescope control law, we can get a ball-park feel for the problem by assuming that the telescope velocity is linearly decreasing from max to target velocity.

Thus, target av. vel = 1.135 and telescope av. vel = $(1.333 + 1.135)/2 = 1.234$ so the velocity differential is 0.099 deg/sec.

The time to catch up 4.27 deg is now 43.13 sec

Therefore, total time to catch up with matching velocity = 64 sec

Note that this is very close to the result observed during testing on the telescope.