



# MULTIPLE MIRROR TELESCOPE OBSERVATORY

Smithsonian Astrophysical Observatory and Steward Observatory, University of Arizona

## M.M.T.O. INTERNAL TECHNICAL MEMORANDUM 88-1

From: A. Poyner

Subject: Software Limits on Telescope Performance: Considerations for  
Catching and Tracking a Fast Satellite

Date: June 15, 1988

### Software Limits on Acceleration and Velocity

	<u>AZ</u>	<u>EL</u>
Max Jerk	0.001073 deg/sec <sup>3</sup>	0.001073 deg/sec <sup>2</sup>
Max Acceleration	0.064373 deg/sec <sup>2</sup>	0.064373 deg/sec <sup>2</sup>
Max Accel achieved in	0.15 sec	0.15 sec
Max Velocity	1.33338 deg/sec	1.50 deg/sec
Max Velocity achieved in	20.8 sec	23.4 sec

(See later section for derivation of these figures.)

NOTE: All MAX VELOCITIES given are PEAK. They are reduced at high/low elevations. Acceleration and jerk maximums are constant, however.

### For Satellite Tracking

For satellite tracking from a catalog of AZ/EL coordinates, at the instant of the starting catalog entry, even if the telescope is at the right position, the target will have some velocity and the telescope will be at rest. For fast satellites, the telescope may not be able to catch it until near the end of the trajectory. Thus, it is necessary to extrapolate the catalog back to some calculated telescope starting point in order to allow the telescope to achieve target velocity at the start of the real catalog. This study is the result of an unsuccessful attempt to track a satellite whose coordinates are listed in an appendix. Figures from this catalog are used in the following examples.

Time to achieve a velocity of V deg/sec is:

$$T = \frac{V}{A_{\text{Max}}} + \frac{1}{2} * \frac{A_{\text{Max}}}{J_{\text{Max}}}$$
$$= \underline{15.534 * V + 0.075} \text{ secs.}$$

For our example case,  $V_{\text{AZ}} = 0.707 \text{ deg/sec}$  and  $V_{\text{EL}} = 0.273 \text{ deg/sec}$ .

So, in this instance, AZ requires the most time to achieve initial velocity.

Using the above equation,  $T = \underline{11.06 \text{ sec}}$

Therefore, extrapolating back about 15 sec should be safe.

To have target velocity achieved at the first of the real coordinates (match velocities at the FIRST coordinate, because you cannot start at the first coordinate and have expectations of matching it at the second), we have to extrapolate back two sets of coordinates. (Remember, satellite tracking starts at the second point in the catalog, not the first.)

Thus, make the two sets of coordinates spaced back by the extrapolation time you calculated, at a position assuming velocity constant at half the target velocity (this assumes constant acceleration to the target velocity, which is close enough).

Therefore, if you have target velocity  $V$  at the first satellite coordinate (time  $T_0$ ), and have decided to extrapolate back by  $T$  seconds, then:

Coordinate 1	Time = $T_0 - 2*T$ Posn = $P_0 - 2*T * V/2$
Coordinate 2	Time = $T_0 - T$ Posn = $P_0 - T * V/2$
Coordinate 3	Time = $T_0$ (First actual satellite position.) Posn = $P_0$ (Vel = $V$ )

In our example,  $T = 15 \text{ sec.}$ ,  $V_{AZ} = 0.707 \text{ deg/sec.}$ , and  $V_{EL} = 0.273 \text{ deg/sec.}$

Thus:

Coordinate 1	Time = $T_0 - 30 \text{ sec.}$ AZ = $-40.48047 - 10.605 = -51.0855 \text{ deg}$ EL = $32.36883 - 4.095 = 28.2738 \text{ deg}$
Coordinate 2	Time = $T_0 - 15 \text{ sec.}$ AZ = $-40.48047 - 5.303 = -45.7835 \text{ deg}$ EL = $+32.36883 - 2.047 = 30.3218 \text{ deg}$

and Coordinate 3 is the first coordinate of the original catalog.

### Why We Couldn't Catch It

Without extrapolating back, we tried to catch and track the satellite using the catalog shown in the appendix. We actually caught up with it about 60 seconds into its 80 second pass. This is why:

We started at T0 with Telescope Velocity = 0 and Target Velocity = 0.707.

The acceleration phase of the telescope to max AZ velocity takes 21 seconds, by which time the target velocity was 1.05, and during which the following equations apply (approximately):

$$S_{tel} = (1/2) * a * t^2$$

$$S_{targ} = V * t \quad (V = \text{average velocity, near enough})$$

where S is angular distance travelled. Thus, at the end of 21 seconds

$$S_{tel} = 1/2 * 0.064 * 21 * 21 = 14.11 \text{ deg}$$

$$S_{targ} = (0.7 + 1.05)/2 * 21 = 18.38 \text{ deg}$$

Therefore, at the point where the telescope achieves maximum velocity, the target is leading the telescope by 4.27 deg.

At constant telescope max velocity of 1.333 deg/sec., and average target velocity over the next 20 - 40 seconds of approx  $(1.05 + 1.22)/2 = 1.135$ , the velocity differential is about 0.2 deg/sec.

The time to catch up 4.27 deg at 0.2 deg/sec = 21.35 sec

Therefore, total time to catch up = 42.35 sec

Note, this is the minimum time to catch the target, with no attempt to match velocities at the interception point, and assuming maximum achievable telescope velocity all the way. This is not the way the telescope control law works, of course. It will bring velocity error and position error to zero at the same point in time. Without trying to simulate the telescope control law, we can get a ball-park feel for the problem by assuming that the telescope velocity is linearly decreasing from max to target velocity.

Thus, target av. vel = 1.135 and telescope av. vel =  $(1.333 + 1.135)/2 = 1.234$  so the velocity differential is 0.099 deg/sec.

The time to catch up 4.27 deg is now 43.13 sec

Therefore, total time to catch up with matching velocity = 64 sec

Note that this is very close to the result observed during testing on the telescope.

Appendix 1: Calculations used to derive telescope performance

Basic equations of motion:

$$S = U*t + 1/2 a*t^2$$

$$V = U + a*t$$

$$v^2 = U^2 + 2*a*S$$

Telescope motion is non-linear; acceleration will increase at the rate of max jerk until max acceleration is reached, the telescope will then accelerate at max acceleration until max velocity is achieved.

Thus:

$$a = J_{Max} * t \quad \text{for } 0 < a < A_{Max} \quad (\text{where } J_{Max} = \text{max jerk})$$

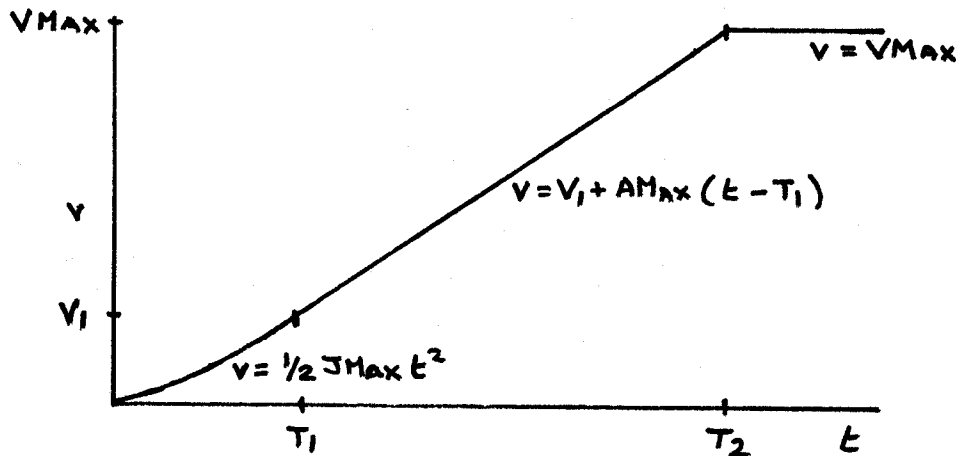
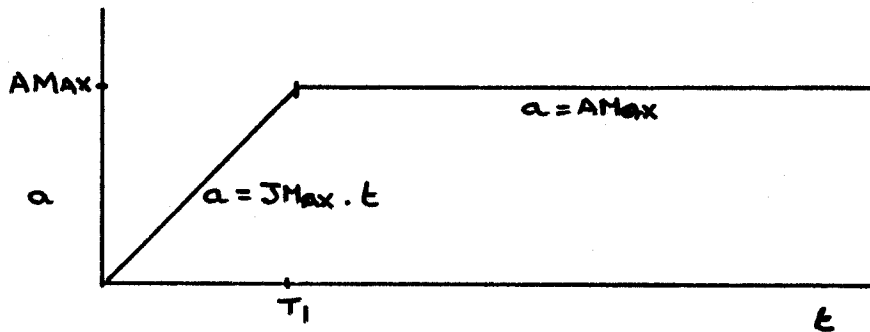
$$a = A_{Max} \quad \text{thereafter}$$

$$v = 1/2 J_{Max} * t^2 \quad \text{for } 0 < a < A_{Max}$$

$$v = V_1 + A_{Max} * (t - T_1) \quad \text{for } V_1 < v < V_{Max} \quad (\text{where } V_1, T_1 \text{ occur when } A_{Max} \text{ is achieved})$$

$$v = V_{Max} \quad \text{thereafter} \quad (V_{Max} \text{ achieved at time } T_2)$$

Graphically:



Using the equations of motion:

$$T_1 = \frac{A_{Max}}{J_{Max}}$$

$$V_1 = \frac{1}{2} J_{Max} * \frac{A_{Max}^2}{J_{Max}^2} = \frac{1}{2} \frac{A_{Max}^2}{J_{Max}}$$

$$T_2 = \frac{V_{Max} - V_1 + T_1 * A_{Max}}{A_{Max}} = \frac{V_{Max}}{A_{Max}} + \frac{1}{2} \frac{A_{Max}}{J_{Max}}$$

Values from the mount program are:

	AZ	E1
A <sub>Max</sub>	3000	3000
J <sub>Max</sub>	1000	1000
V <sub>Max</sub>	1242800	1398100

These values are used to calculate 100V in the mount program, so must be divided by 100 before we use them in our equations. The unit of time in these numbers is one control interval, or 1/20 sec, and the unit of angular distance is the encoder unit, or (360/2<sup>26</sup>) degrees (or 1/186413.5 degrees).

Thus:

$$\text{Max Jerk} = 10.00 * (360/2^{26}) / (1/20)^3 \text{ deg/sec}^3$$

$$= \underline{0.001073 \text{ deg/sec}^3}$$

$$\text{Max acceleration} = 30.00 * (360/2^{26}) / (1/20)^2 \text{ deg/sec}^2$$

$$= \underline{0.064373 \text{ deg/sec}^2}$$

$$\text{Max accel achieved at } T_1 = \frac{30.00 * 1 \text{ sec}}{10.00 * 20}$$

$$= \underline{0.15 \text{ sec}}$$

$$\text{Max velocity (AZ)} = 12428.00 * (360/2^{26}) / (1/20)$$

$$= \underline{1.33338 \text{ deg/sec}}$$

$$\text{Max vel (AZ) achieved at } T_2 = \frac{(12428.00 + 30.00) * 1}{30.00 * 20.00 * 20}$$

$$= \underline{20.8 \text{ sec}}$$

$$\text{Max velocity (EL)} = 13981.00 * (360/2^{26}) / (1/20)$$

$$= \underline{1.50 \text{ deg/sec}}$$

$$\text{Max vel (EL) achieved at } T_2 = \frac{(13981.00 + 30.00) * 1}{30.00 * 20.00 * 20}$$

$$= \underline{23.4 \text{ sec}}$$

## Appendix 2: Hardware Limits on Telescope Performance

To be sure that the limiting factors on telescope performance are being applied by software and not by mechanical limitations, I asked Dan Bianco for an evaluation. He produced the following figures. They are strictly "first order" calculations, but they are realistic ball-park figures, which show that the telescope is capable of much higher acceleration than the software limits.

MMT Maximum acceleration:

EL axis inertia: 177,000 lb.ft.sec<sup>2</sup>

AZ axis inertia: 681,000 lb.ft.sec<sup>2</sup>

EL Torque available: 4188 ft.lb.

AZ Torque available: 8356 ft.lb.

EL Friction torque: 240 ft.lb.

AZ Friction torque: 280 ft.lb.

Motor  $K_T$  = 2.46 ft.lb./amp x 18 amps max  
= 44.28 ft.lb. per motor

2 motor = 88.56 ft.lb x 100:1 gearing

which gives (8856 - 280), or 8576 ft.lb. above torque (AZ)

and 4188 ft.lb. (EL)

Acceleration: (neglecting damping)

$$= T/I$$

$$(AZ) = \frac{4183 \text{ ft.lb}}{177000 \text{ lb.ft.s}^2} = .0237 \text{ rad/sec}^2 = \underline{1.358 \text{ deg/sec}^2}$$

$$(EL) = \frac{8856}{681000} = .013 \text{ rad/sec}^2 = \underline{0.745 \text{ deg/sec}^2}$$

Maximum motor acceleration: 420 rad/sec<sup>2</sup>

### Appendix 3: VMax Envelopes

In both axes, the value of VMax at any time depends on elevation angle. In the preceding discussions we used the peak value of VMax, and one should be aware that this may not always be the case.

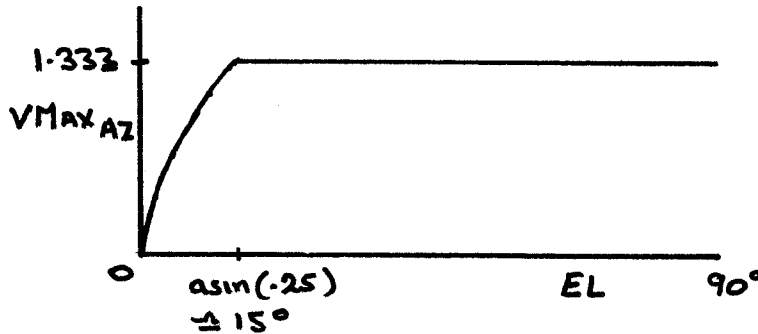
#### Azimuth VMax

For azimuth, maximum velocity is reduced at very low angles of elevation, to avoid the possibility of damage to optics if an AZ slew was abruptly stopped while the telescope was down near the horizon. This limit reduction applies only below about 15 degrees elevation.

$$V_{MAX_{AZ}} = 4 * \sin(EL) * V_{MAX_{PEAK}} \quad \text{for } 0 < EL < \text{asin}(0.25)$$

$$V_{MAX_{AZ}} = V_{MAX_{PEAK}} \quad \text{for } \text{asin}(0.25) < EL < 90$$

Graphically:



#### Elevation VMax

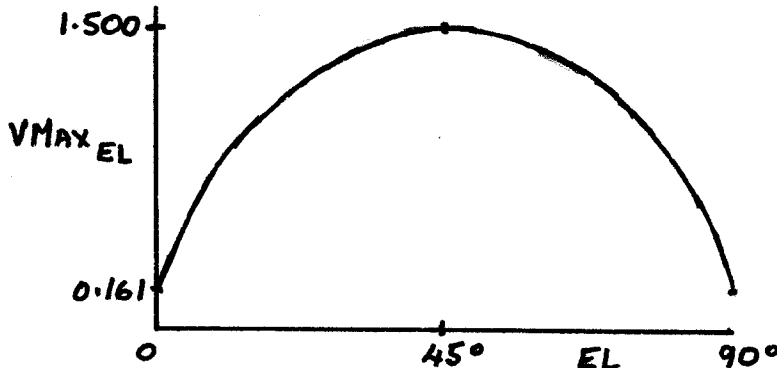
Elevation maximum velocity is limited by a sine function, from a minimum value at 0 and 90 degrees to a maximum value at 45 degrees.

$$V_{MAX_{EL}} = \sin(2 * EL) * \$bvmax + \$avmax$$

From the mount program:     \$avmax = 1500  
                                   \$bvmax = 12481

giving a peak value of 1.500 deg/sec at 45 degrees, and  
min. value of 0.161 deg/sec at 0 and 90 degrees.

Graphically:



# APPENDIX 4: TEST SATELLITE CATALOG

NOTE - CIRCLED POINTS ENTERED IN MMT CATALOG.

selected satellite altitude was 300 Km  
 The satellite velocity = 7.681422 km/sec  
 the closest pt of approach is 360 Km

time (sec)	az ang (deg)	az rate (deg/sec)	az accel (deg/s <sup>2</sup> )	el ang (deg)	el rate (deg/s)	el acc (deg/s <sup>2</sup> )
-40.00000	-40.48047	0.70730	0.01490	32.36883	0.27297	0.00016
-39.00000	-39.76567	0.72233	0.01516	32.64182	0.27301	-0.00007
-38.00000	-39.03572	0.73761	0.01540	32.91477	0.27282	-0.00032
-37.00000	-38.29037	0.75313	0.01563	33.18739	0.27237	-0.00058
-36.00000	-37.52939	0.76887	0.01585	33.45943	0.27166	-0.00085
-35.00000	-36.75256	0.78483	0.01606	33.73063	0.27067	-0.00113
-34.00000	-35.95967	0.80098	0.01625	34.00067	0.26939	-0.00144
-33.00000	-35.15054	0.81731	0.01642	34.26929	0.26780	-0.00175
-32.00000	-34.32499	0.83381	0.01657	34.53616	0.26588	-0.00208
-31.00000	-33.48287	0.85045	0.01670	34.80094	0.26363	-0.00243
-30.00000	-32.62406	0.86720	0.01680	35.06329	0.26103	-0.00278
-29.00000	-31.74844	0.88404	0.01688	35.32287	0.25806	-0.00315
-28.00000	-30.85595	0.90095	0.01693	35.57930	0.25471	-0.00354
-27.00000	-29.94653	0.91789	0.01694	35.83216	0.25098	-0.00393
-26.00000	-29.02017	0.93483	0.01692	36.08111	0.24684	-0.00434
-25.00000	-28.07689	0.95172	0.01687	36.32571	0.24229	-0.00476
-24.00000	-27.11675	0.96854	0.01677	36.56555	0.23732	-0.00518
-23.00000	-26.13984	0.98525	0.01663	36.80021	0.23192	-0.00562
-22.00000	-25.14631	1.00179	0.01644	37.02925	0.22609	-0.00606
-21.00000	-24.13634	1.01812	0.01621	37.25223	0.21981	-0.00650
-20.00000	-23.11015	1.03420	0.01593	37.46874	0.21309	-0.00695
-19.00000	-22.06805	1.04997	0.01560	37.67827	0.20592	-0.00739
-18.00000	-21.01034	1.06538	0.01522	37.88042	0.19831	-0.00784
-17.00000	-19.93742	1.08038	0.01478	38.07473	0.19025	-0.00828
-16.00000	-18.84973	1.09492	0.01429	38.26077	0.18175	-0.00871
-15.00000	-17.74775	1.10894	0.01374	38.43809	0.17283	-0.00914
-14.00000	-16.63204	1.12238	0.01314	38.60627	0.16348	-0.00955
-13.00000	-15.50320	1.13519	0.01248	38.76492	0.15373	-0.00995
-12.00000	-14.36188	1.14732	0.01177	38.91361	0.14358	-0.01034
-11.00000	-13.20881	1.15870	0.01100	39.05197	0.13306	-0.01070
-10.00000	-12.04474	1.16930	0.01018	39.17960	0.12218	-0.01105
-9.00000	-10.87049	1.17906	0.00932	39.29621	0.11097	-0.01137
-8.00000	-9.68692	1.18792	0.00841	39.40145	0.09946	-0.01166
-7.00000	-8.49495	1.19586	0.00746	39.49503	0.08766	-0.01192
-6.00000	-7.29553	1.20282	0.00647	39.57670	0.07562	-0.01216
-5.00000	-6.08965	1.20878	0.00544	39.64620	0.06336	-0.01236
-4.00000	-4.87832	1.21370	0.00439	39.70335	0.05091	-0.01253
-3.00000	-3.66261	1.21755	0.00331	39.74798	0.03832	-0.01266
-2.00000	-2.44359	1.22031	0.00222	39.77993	0.02561	-0.01275
-1.00000	-1.22235	1.22198	0.00111	39.79915	0.01282	-0.01281
0.00000	0.00000	1.22254	0.00000	39.80558	0.00000	-0.01283

1.00000	1.22235	1.22198	-0.00111	39.79915	-0.01282	-0.01281
2.00000	2.44359	1.22031	-0.00222	39.77993	-0.02561	-0.01275
3.00000	3.66261	1.21755	-0.00331	39.74798	-0.03832	-0.01266
4.00000	4.87832	1.21370	-0.00439	39.70335	-0.05091	-0.01253
5.00000	6.08965	1.20878	-0.00544	39.64620	-0.06336	-0.01236
6.00000	7.29553	1.20282	-0.00647	39.57670	-0.07562	-0.01216
7.00000	8.49495	1.19586	-0.00746	39.49503	-0.08766	-0.01192
8.00000	9.68692	1.18792	-0.00841	39.40145	-0.09946	-0.01166
9.00000	10.87049	1.17906	-0.00932	39.29621	-0.11097	-0.01137
10.00000	12.04474	1.16930	-0.01018	39.17960	-0.12218	-0.01105
11.00000	13.20881	1.15870	-0.01100	39.05197	-0.13306	-0.01070
12.00000	14.36188	1.14732	-0.01177	38.91361	-0.14358	-0.01034
13.00000	15.50320	1.13519	-0.01248	38.76492	-0.15373	-0.00995
14.00000	16.63204	1.12238	-0.01314	38.60627	-0.16348	-0.00955
15.00000	17.74775	1.10894	-0.01374	38.43809	-0.17283	-0.00914
16.00000	18.84973	1.09492	-0.01429	38.26077	-0.18175	-0.00871
17.00000	19.93742	1.08038	-0.01478	38.07473	-0.19025	-0.00828
18.00000	21.01034	1.06538	-0.01522	37.88042	-0.19831	-0.00784
19.00000	22.06805	1.04997	-0.01560	37.67827	-0.20592	-0.00739
20.00000	23.11015	1.03420	-0.01593	37.46874	-0.21309	-0.00695
21.00000	24.13634	1.01812	-0.01621	37.25223	-0.21981	-0.00650
22.00000	25.14631	1.00179	-0.01644	37.02925	-0.22609	-0.00606
23.00000	26.13984	0.98525	-0.01663	36.80021	-0.23192	-0.00562
24.00000	27.11675	0.96854	-0.01677	36.56555	-0.23732	-0.00518
25.00000	28.07689	0.95172	-0.01687	36.32571	-0.24229	-0.00476
26.00000	29.02017	0.93483	-0.01692	36.08111	-0.24684	-0.00434
27.00000	29.94653	0.91789	-0.01694	35.83216	-0.25098	-0.00393
28.00000	30.85595	0.90095	-0.01693	35.57930	-0.25471	-0.00354
29.00000	31.74844	0.88404	-0.01688	35.32287	-0.25806	-0.00315
30.00000	32.62406	0.86720	-0.01680	35.06329	-0.26103	-0.00278
31.00000	33.48287	0.85045	-0.01670	34.80094	-0.26363	-0.00243
32.00000	34.32499	0.83381	-0.01657	34.53616	-0.26588	-0.00208
33.00000	35.15054	0.81731	-0.01642	34.26929	-0.26780	-0.00175
34.00000	35.95967	0.80098	-0.01625	34.00067	-0.26939	-0.00144
35.00000	36.75256	0.78483	-0.01606	33.73063	-0.27067	-0.00113
36.00000	37.52939	0.76887	-0.01585	33.45943	-0.27166	-0.00085
37.00000	38.29037	0.75313	-0.01563	33.18739	-0.27237	-0.00058
38.00000	39.03572	0.73761	-0.01540	32.91477	-0.27282	-0.00032
39.00000	39.76567	0.72233	-0.01516	32.64182	-0.27301	-0.00007
40.00000	40.48047	0.70730	-0.01490	32.36883	-0.27297	0.00016