MULTIPLE MIRROR TELESCOPE OBSERVATORY

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MMTO TECHNICAL MEMORANDUM 92-1

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SUBJECT: Coordinate Transformations at the M.M.T.O.

DATE: February 14, 1992

Coordinate transformations at the MMTO are used to convert from Right Ascension and Declination coordinates of astronomical objects to Elevation and Azimuth coordinates required by the telescope, and back again. The third telescope axis, rotator angle, is a function of Parallactic Angle, calculated from the RA and Dec coordinates of the object, as well as any desired offset given by the observer. All transformations also incorporate Local Sidereal Time, used to calculate the Hour Angle of the object.

In brief, the equations used for these transformations are given below.

$$HA = LST - RA$$

$$\sin(El) = \sin(Dec)\sin(Lat) + \cos(Dec)\cos(HA)\cos(Lat)$$

$$\tan(Az) = \frac{-\cos(Dec)\sin(HA)}{\sin(Dec)\cos(Lat) - \cos(Dec)\cos(HA)\sin(Lat)}$$

$$\sin(Dec) = \sin(El)\sin(Lat) + \cos(El)\cos(Az)\cos(Lat)$$

$$\tan(HA) = \frac{-\cos(El)\sin(Az)}{\sin(El)\cos(Lat) - \cos(El)\cos(Az)\sin(Lat)}$$

$$\tan(PA) = \frac{-\cos(Lat)\sin(HA)}{\sin(Lat)\cos(Dec) - \cos(Lat)\cos(HA)\sin(Dec)}$$

$$\text{Rotator Angle} = PA + \text{Rotator Offset} + \text{Sky Offset}$$

The derivations of these equations, and the calculations of the axial velocities, are given in the attached paper. Some of these relationships are difficult to visualize, so Morris Kaufman has produced graphs to help minimize this problem.

Coordinate Transformations at M.M.T.O

MMTO TECHNICAL MEMORANDUM 92-1

by Anthony D. Poyner

with graphics by Morris Kaufman

1.0 Symbols and definitions

- **R** Right Ascension, measured from the equinox Eastwards in the plane of the equator.
- **D** Declination, measured perpendicular to the equator, positive to North.
- **H** Hour Angle, measured Westwards in the plane of the of the equator from the local meridian.
- **A** Azimuth, measured from North through East in the plane of the horizon.
- **E** Elevation (equal to 90 - Zenith Distance), measured from horizon to zenith.
- **P** Parallactic Angle, measured from North through East. In other words, positive PA is CCW when viewed from below the rotator. NOTE that this is the opposite sense from the standard references, which measure PA in the same sense as HA. We apparently find it more convenient to think of PA in the same sense as AZ.
- **L** Latitude (equals 31:41:19.7 at the MMT), measured perpendicular to the ecliptic, positive to North.
- **LNG** Longitude (equals 110:53:04.4 at the MMT - quoted from *Astronomical Almanac.* Note that the old Mount program used 110:53:29.1, which gives a 1.6 seconds of time difference to LST, presumably removed by pointing corrections.) Measured from the equinox eastwards in the plane of the ecliptic (a strange way of putting it - means measured positive to the West of the Greenwich Meridian).
- **LST** Local Sidereal Time, the hour angle of the vernal equinox, equal to Greenwich meridian sidereal time - longitude.

The definitions above are based on definitions in the *Explanatory Supplement.*

2.0 General Equations

The following equations 1 - 7 are detailed in *Lang, "Astrophysical Formulae."*

For transformations between Alt/Az and HA/Dec (or RA/Dec):

$$\cos(E)\sin(A) = -\cos(D)\sin(H) \quad (1)$$

$$\cos(E)\cos(A) = \sin(D)\cos(L) - \cos(D)\cos(H)\sin(L) \quad (2)$$

$$\sin(E) = \sin(D)\sin(L) + \cos(D)\cos(H)\cos(L) \quad (3)$$
$$\cos(D)\cos(H) = \sin(E)\cos(L) - \cos(E)\cos(A)\sin(L) \quad (4)$$
$$\sin(D) = \sin(E)\sin(L) + \cos(E)\cos(A)\cos(L) \quad (5)$$
$$H = LST - R \quad (6)$$

and for parallactic angle:

$$\sin(P)\cos(E) = -\cos(L)\sin(H) \quad (7)$$

(Reversed sign gives PA in same direction as AZ)

From Eqn 1: $\frac{\cos(E)}{-\sin(H)} = \frac{\cos(D)}{\sin(A)}$

From Eqn 7: $\frac{\cos(E)}{-\sin(H)} = \frac{\cos(L)}{\sin(P)}$

giving us: $\frac{\cos(E)}{-\sin(H)} = \frac{\cos(D)}{\sin(A)} = \frac{\cos(L)}{\sin(P)}$ (8)

From this useful relationship, we find that Eqns 1-5 and 7 can be translated to other forms by substituting E<>D and A<>-H; or E<>L and -H<>P; or D<>L and A<>P, in these equations.

3.0 Equations used in Mount Program

3.1 AZ/EL Calculations

Elevation is calculated from Eqn 3, (range 0 to 90, thus no ambiguity).

$$\sin(E) = \sin(D)\sin(L) + \cos(D)\cos(H)\cos(L)$$

Azimuth has a range +180 to -180, thus has ambiguity. We resolve this by using the atan2 function with the following arguments:

$\sin(A)\cos(E)$ from Eqn 1, and:
$\cos(A)\cos(E)$ from Eqn 2, thus

$$\tan(A) = \frac{-\cos(D)\sin(H)}{\sin(D)\cos(L) - \cos(D)\cos(H)\sin(L)}$$

3.2 RA/DEC Calculations

Similarly, Declination is calculated from Eqn 5, (range -90 to 90, thus no ambiguity).

$$\sin(D) = \sin(E)\sin(L) + \cos(E)\cos(A)\cos(L)$$

Hour Angle has a range +12 to -12 hours, thus has ambiguity. We resolve this by using the atan2 function with the following arguments:

$\sin(H)\cos(D)$ from Eqn 1, and:
$\cos(H)\cos(D)$ from Eqn 4, thus

$$\tan(H) = \frac{-\cos(E)\sin(A)}{\sin(E)\cos(L) - \cos(E)\cos(A)\sin(L)}$$

And Right Ascension, from Eqn 6:

$$R = LST - H$$

3.3 Parallactic and Rotator Angle Calculations

To calculate Parallactic Angle, we use Eqn 7 and re-write Eqn 2, using the relationships of Eqn 8 (substituting A<>P, D<>L), to give:

$$\sin(P)\cos(E) = -\cos(L)\sin(H)$$
$$\cos(P)\cos(E) = \sin(L)\cos(D) - \cos(L)\cos(H)\sin(D) \qquad (9)$$

$$\tan(P) = \frac{-\cos(L)\sin(H)}{\sin(L)\cos(D) - \cos(L)\cos(H)\sin(D)}$$

Rotator angle is then calculated as:

$$\text{Rotator Angle} = P + \text{Rotator Offset} + \text{Sky Offset}$$

where:

Rotator Offset is an operator-entered value of rotator angle required to align the instrument slit (or equivalent) North - South. Usually zero for MMT instruments.

Sky Offset is an operator-entered value of rotator angle required to align the instrument slit along some feature of the object to be observed. This is usually referred to by astronomers as Position Angle, and increases from North through East.

4.0 Velocities

By manipulating and differentiating equations 1 - 7, we can calculate the velocities of the three telescope axes. In the following sections, the character ′ is used to indicate first derivative with respect to time.

4.1 Elevation Velocity

Differentiating Eqn 3 gives:

$$\cos(E).E' = -\cos(D)\cos(L)\sin(H).H'$$

or

$$E' = \frac{-\cos(D)\cos(L)\sin(H)}{\cos(E)}.H'$$

Substituting for -sin(H)/cos(E) from Eqn 7 gives:

$$E' = \frac{\cos(D)\cos(L)\sin(A)}{\cos(D)}.H'$$

or

$$E' = H'\cos(L)\sin(A)$$

As H′ is 15 arcsec/sec and cos(L) is approximately 0.851 at the MMT, we see that

Max Elevation Tracking Velocity = 12.8 arcsec/sec @ AZ = 90, 270.
Min Elevation Tracking Velocity = 0.0 arcsec/sec @ AZ = 0, 180.

4.2 Azimuth Velocity

From Eqn 7, substituting P<>A and D<>L gives:

$$\sin(A)\cos(E) = -\cos(D)\sin(H)$$

and differentiating:

$$-\sin(A)\sin(E).E' + \cos(E)\cos(A).A' = -\cos(D)\cos(H).H'$$

or

$$A' = \frac{\sin(A)\sin(E).E' - \cos(D)\cos(H).H'}{\cos(E)\cos(A)}$$

Substituting for E′, calculated in section 4.1, gives:

$$A' = \frac{\sin^2(A)\sin(E)\cos(L).H' - \cos(D)\cos(H).H'}{\cos(E)\cos(A)}$$

Substituting for cos(D)cos(H) from Eqn 4, gives:

$$A' = H'\{\frac{\sin^2(A)\sin(E)\cos(L) - \sin(E)\cos(L) + \cos(E)\cos(A)\sin(L)}{\cos(E)\cos(A)}\}$$

$$A' = H'\{\frac{\sin^2(A)\tan(E)\cos(L)}{\cos(A)} - \frac{\tan(E)\cos(L)}{\cos(A)} + \sin(L)\}$$

$$A' = H'\{\frac{-\tan(E)\cos(L)\{1 - \sin^2(A)\}}{\cos(A)} + \sin(L)\}$$

Thus:

$$A' = -H'\{\tan(E)\cos(A)\cos(L) - \sin(L)\}$$

For the MMT, this reduces to (approximately):

$$A' = -12.76\ \tan(E)\cos(A) + 7.88 \quad \text{arcsec/sec}$$

Therefore, azimuth tracking velocity maximum is infinite at EL = 90, and minimum is zero at EL = 0. As the telescope obviously cannot rotate infinitely fast, the maximum physical capability of the structure dictates an area around zenith that we cannot track through. The maximum azimuth velocity is set in software to be 1.3°/second (limited by building speed). The resulting worst-case "cone of avoidance" occurs at elevation:

$$1.3 \times 3600 = 12.76\ \tan(E) + 7.88$$

or, maximum elevation for tracking equals 89°50.6'.

4.3 Rotator Velocity

From Eqn 7, substituting D<>E and -H<>A gives:

$$\cos(D)\sin(P) = \cos(L)\sin(A)$$

and differentiating:

$$\cos(D)\cos(P).P' = \cos(L)\cos(A).A' \qquad (10)$$

From Eqn 4, substituting -H<>P and E<>L, we get:

$$\cos(D)\cos(P) = \sin(L)\cos(E) - \cos(L)\cos(A)\sin(E) \qquad (11)$$

Substituting (11) into (10), and substituting for A' from the previous section:

$$P' = \frac{\cos(L)\cos(A)\{\sin(L)-\tan(E)\cos(A)\cos(L)\}}{\sin(L)\cos(E) - \cos(L)\cos(A)\sin(E)}.H'$$

or

$$P' = \frac{\cos(L)\cos(A)\{\sin(L)-\tan(E)\cos(A)\cos(L)\}}{\cos(E)\{\sin(L) - \cos(L)\cos(A)\tan(E)\}}.H'$$

Thus:

$$P' = H'\frac{\cos(L)\cos(A)}{\cos(E)}$$

5.0 References

"Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac," HMSO.

"The Astronomical Almanac," U.S. Government Printing Office.

"Astrophysical Formulae," Kenneth R. Lang, Springer-Verlag.

"Computational Spherical Astronomy," Laurence G. Taff, John Wiley & Sons.

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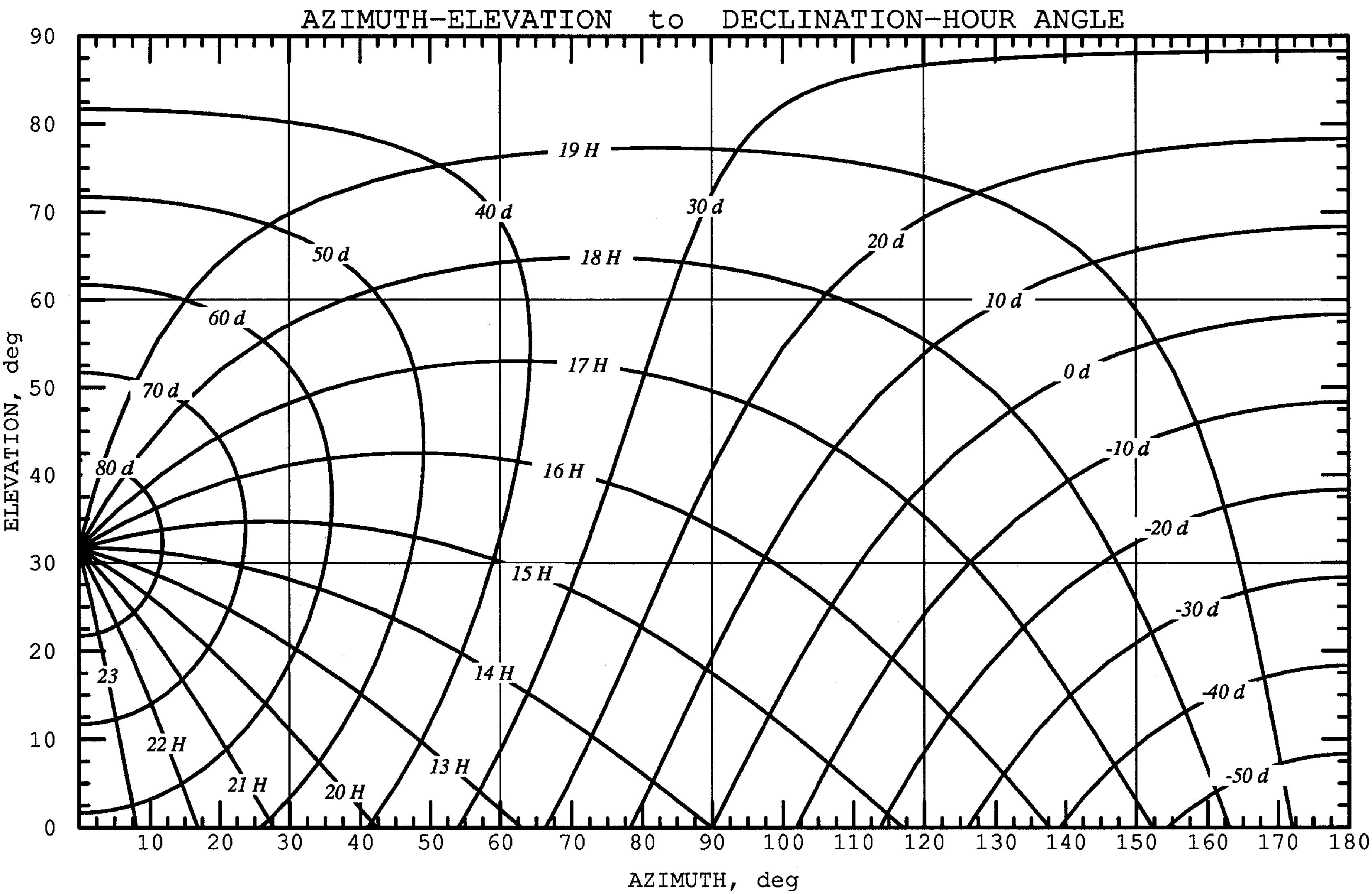

AZIMUTH-ELEVATION to DECLINATION-HOUR ANGLE
ELEVATION, deg
AZIMUTH, deg
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
19 H
18 H
17 H
16 H
15 H
14 H
13 H
20 H
21 H
22 H
23
80 d
70 d
60 d
50 d
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30 d
20 d
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0 d
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-30 d
-40 d
-50 d

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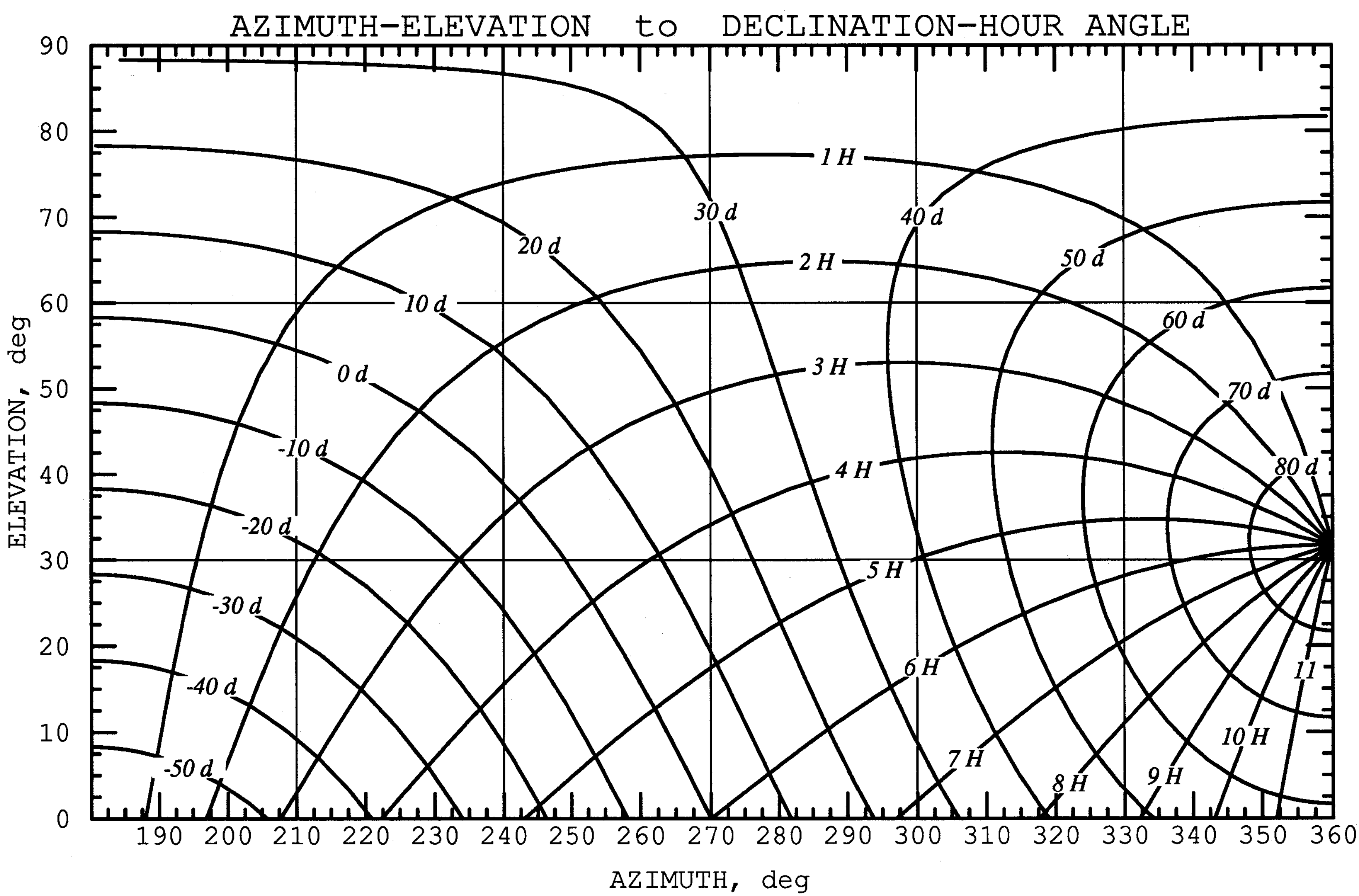
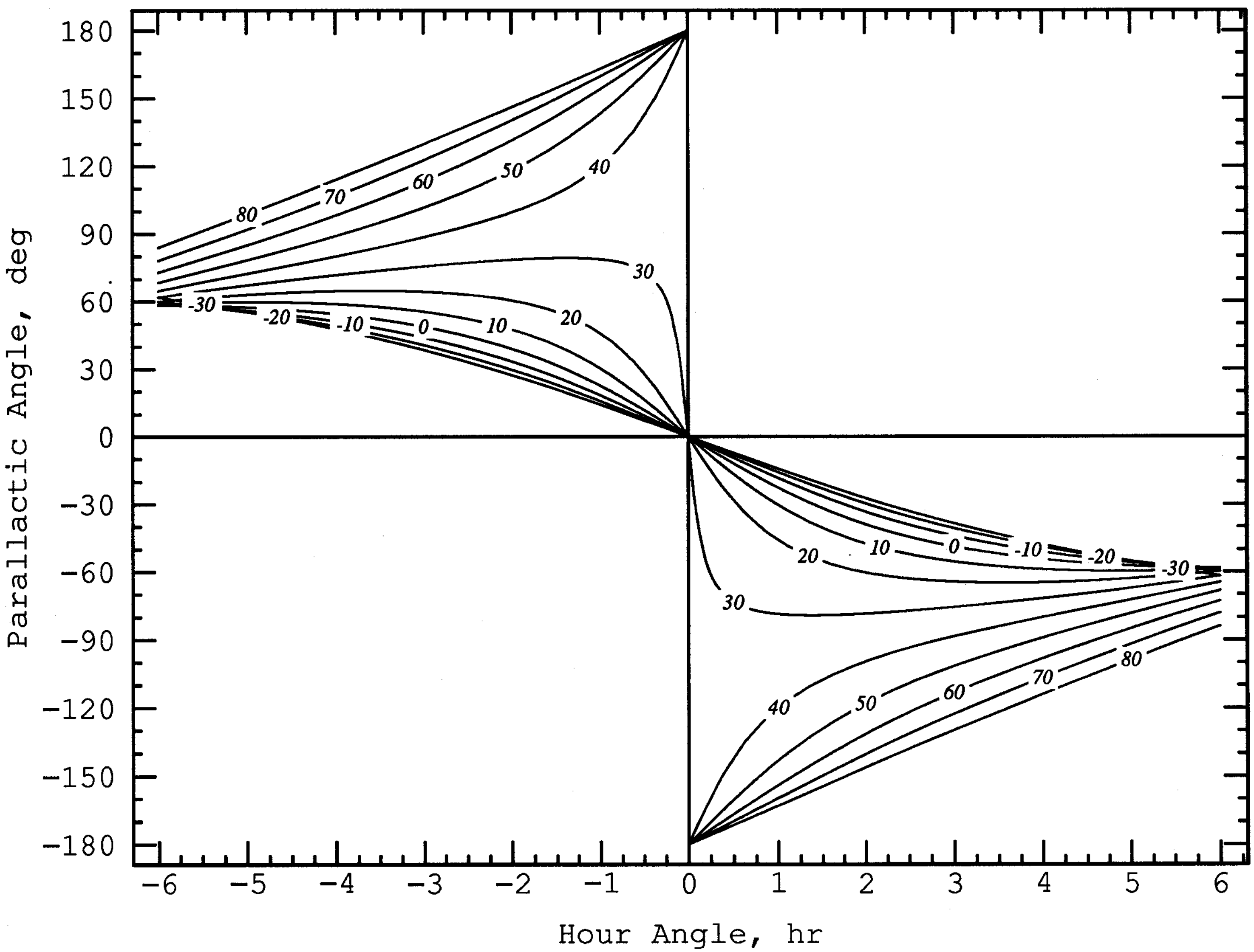
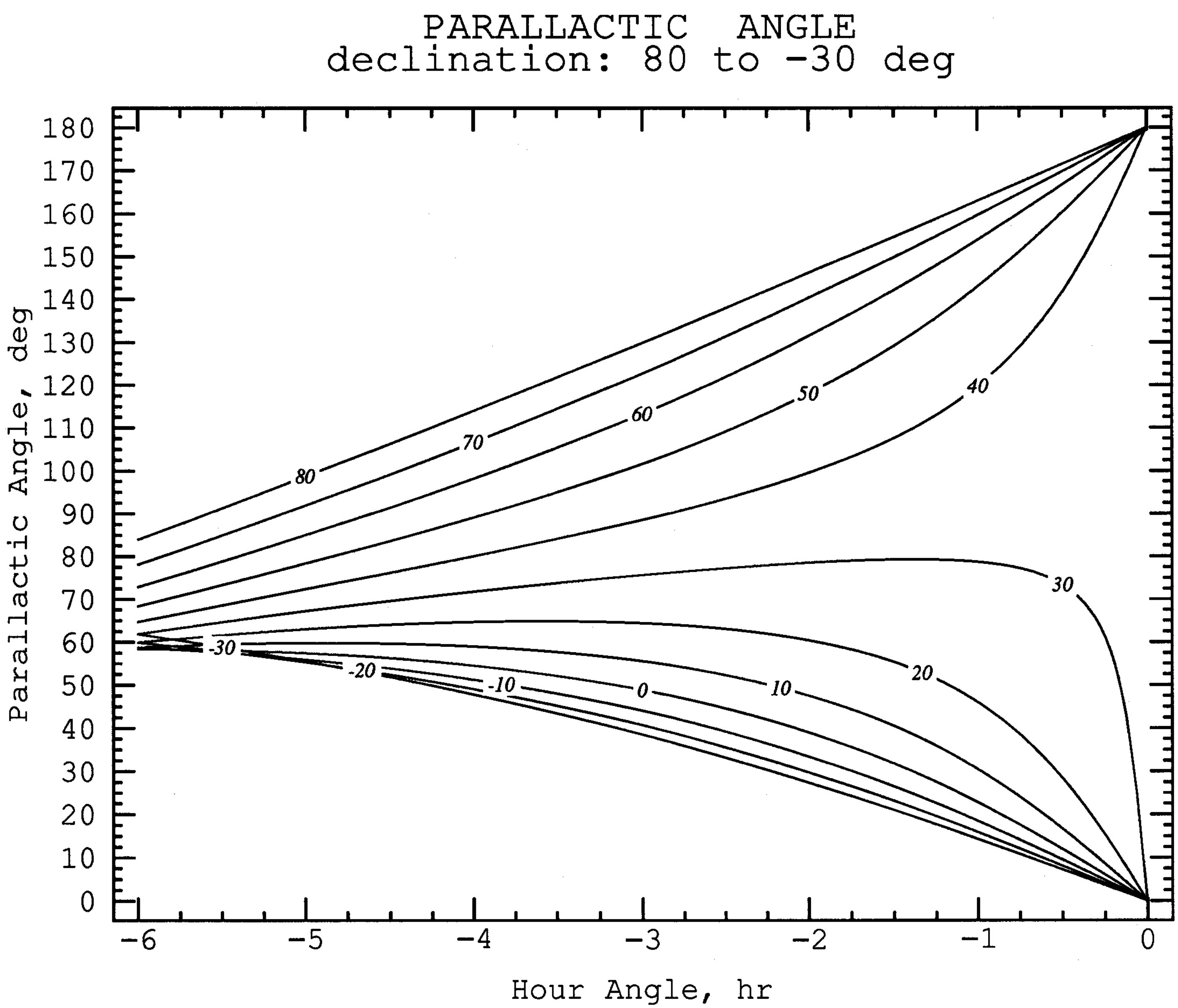

AZIMUTH-ELEVATION to DECLINATION-HOUR ANGLE
ELEVATION, deg
AZIMUTH, deg
0
10
20
30
40
50
60
70
80
90
190
200
210
220
230
240
250
260
270
280
290
300
310
320
330
340
350
360
1 H
2 H
3 H
4 H
5 H
6 H
7 H
8 H
9 H
10 H
11
-50 d
-40 d
-30 d
-20 d
-10 d
0 d
10 d
20 d
30 d
40 d
50 d
60 d
70 d
80 d

PARALLACTIC ANGLE
declination: 80 to −30 deg




PARALLACTIC ANGLE
declination: 80 to -30 deg
Parallactic Angle, deg
Hour Angle, hr
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
-6
-5
-4
-3
-2
-1
0
80
70
60
50
40
30
20
10
0
-10
-20
-30

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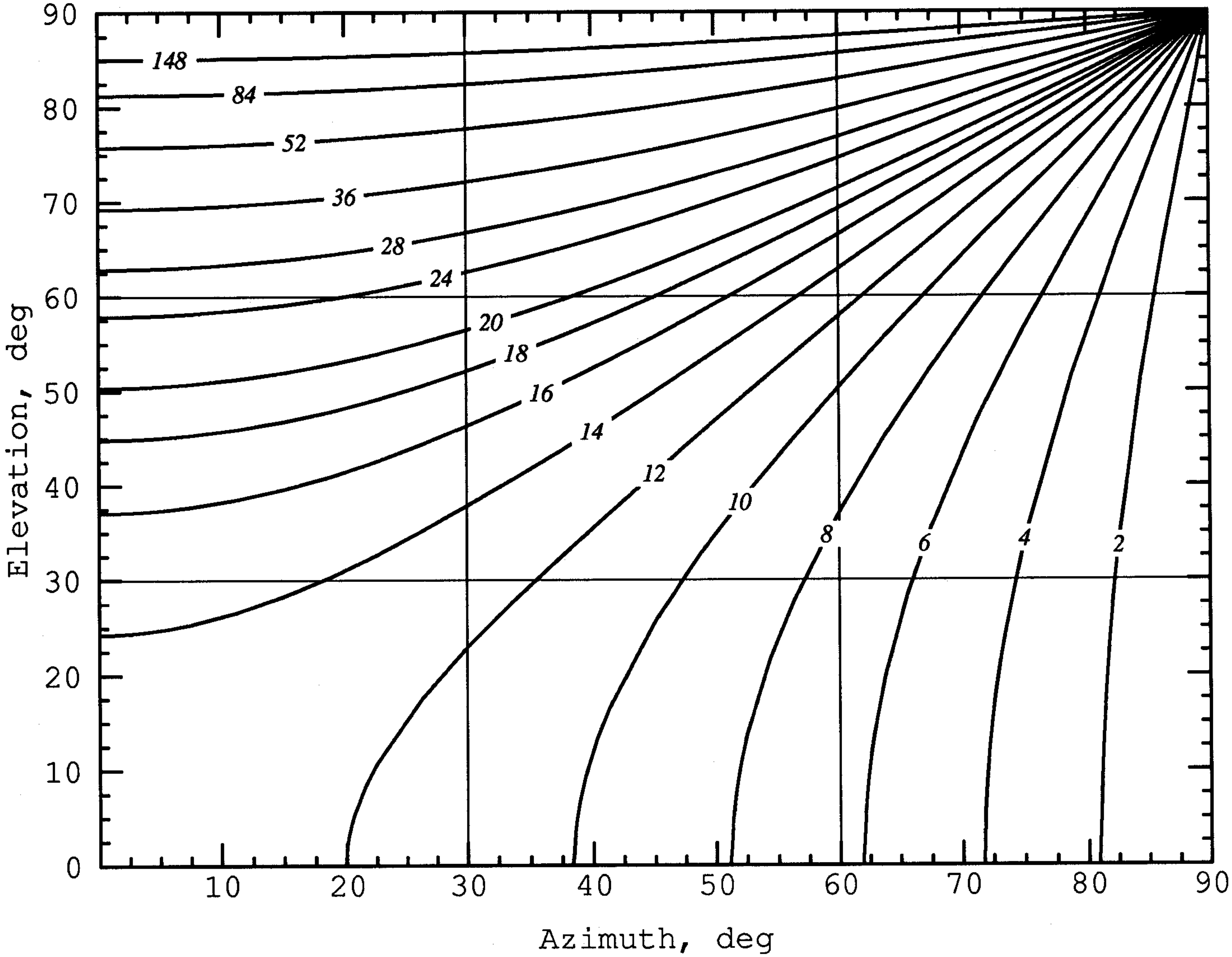
ROTATOR VELOCITY CONTOUR PLOT
velocity in arcsec/sec



MAX ROTATOR VELOCITY vs ZENITH ANGLE