

Technical Report No. 16

Polarimetry on the MMT

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Overview:

Recently, on the 200" at Palomar, it was discovered that BL Lac (16th mag) had polarimetric variations of up to 2 percent in as little as 15 - 30 minutes. One minute integrations on this instrument were sufficient to achieve a polarimetric accuracy of 0.2 percent. However, considering the distance to the telescope, and availability of time to outsiders, it would be useful to carry out observations of this type from Steward.

The 200" collects almost five times as much light as the 90", but only about 1.3 times as much as the MMT. Considering this fact, one finds the MMT has the light gathering capabilities to conduct accurate, short time scale polarimetric observations of BL Lac and other faint objects. The major obstacle to overcome is the effect of instrumental polarizations arising from this instrument. The cause of these, of course, are the oblique reflections of the tertiary and beam combiner.

This report will discuss the possibilities of doing polarimetry on the MMT. As just noted, the large light gathering and resolution capabilities make the study of instrumental polarizations worthwhile. The types and magnitudes of instrumental polarizations will be calculated, then the effectiveness of a simple compensating scheme will be studied.

In order to understand the instrumental effects of the MMT on a given polarization state, it is necessary to understand reflections from a metallic layer.

Once this is accomplished, the MMT can be modelled in terms

of Mueller matrices, allowing instrumental effects to be calculated. It will be assumed that all reflections will occur from aluminium with no overcoat.

Reflection from a metal layer:

Electromagnetic theory describes the phase shifts for the p and s planes (i.e. in the plane of incidence=p, and perpendicular=s), in terms of the index of refraction (n), the absorption coefficient (k), and the angle of incidence (ϕ). The following equations summarize the results (Drude 1959):

$$\tan(\Delta) = \sin(q) * \tan(2p) \quad 1)$$

$$\cos(2X) = \cos(q) * \sin(2p) \quad 2)$$

x

$$\tan(q) = k \quad 3)$$

$$\tan(p) = \frac{n (1 + k^2)^{1/2}}{\sin(\phi) \tan(\phi)} \quad 4)$$

$$\tan(X) = r_p/r_s \quad 5)$$

where: $\Delta = \delta_p - \delta_s$ is the phase difference between the p and s planes.

$\tan(X)$ = ratio of the electric field reflection coefficients for the p and s planes.

Further, the intensity reflection coefficients are given by (Clarke and Grainger):

$$R_p = \frac{(n - \frac{1}{\cos\phi})^2 + k^2}{(n + \frac{1}{\cos\phi})^2 + k^2} = r_p^2 \quad 6)$$

$$R_s = \frac{(n - \cos\phi)^2 + k^2}{(n + \cos\phi)^2 + k^2} = r_s^2 \quad 7)$$

A wave analysis could be conducted using this information,

but it is a simpler matter to express a metallic reflection in terms of Mueller Calculus, because it is readily available and general. The quantities in the above equations can then be calculated for the MMT configuration, and "plugged" into the matrix elements.

It has been found (Borra 1976) that a metallic reflection is equivalently expressed by two polarimetric components--a polarizer and a retarder. Their combination then makes a partially polarizing device (a mirror).

Maintaining this formalism, the polarization states are expressed as Stokes vectors. Thus, for a single reflection:

$$I^{out} = \underline{P(\theta)} \underline{M(p)} I^{in}$$

where the I's are Stokes vectors, $P(\theta)$ is the matrix expression for a polarizer at angle θ , and $M(p)$ is a retarder whose fast axis is rotated to an angle p . Both of these angles are defined with respect to the coordinate system of the Stokes vectors.

For a mirror, the fast axis of the retarder is in the plane of incidence (Borra 1976).

The Mueller matrix for a retarder is (Shurcliff 1962):

$$M(p) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & D^2 + G^2 - E^2 & 2DE & -2EG \\ 0 & 2DE & -D^2 + G^2 + E^2 & 2DG \\ 0 & 2EG & -2DG & 2G^2 - 1 \end{pmatrix} \quad 8)$$

$$D = Q \sin\left(\frac{1}{2}\Delta\right)$$

$$E = U \sin\left(\frac{1}{2}\Delta\right)$$

$$G = \cos\left(\frac{1}{2}\Delta\right)$$

$$U = \sin(2p)$$

$$Q = \cos(2p)$$

hence, the azimuth (p) and the retardance (Δ) express the retarder.

The polarizer is expressed as (Shurcliff 1962, Borra 1976):

$$P(\theta) = \begin{pmatrix} R_s + R_p & C_2(R_s - R_p) & S_2(R_s - R_p) & 0 \\ C_2(R_s - R_p) & C_2^2(R_s + R_p) + 2S_2^2\sqrt{R_s R_p} & C_2 S_2(R_s + R_p) - 2S_2 C_2\sqrt{R_s R_p} & 0 \\ S_2(R_s - R_p) & S_2 C_2(R_s + R_p) - 2S_2 C_2\sqrt{R_s R_p} & S_2^2(R_s + R_p) + 2C_2^2\sqrt{R_s R_p} & 0 \\ 0 & 0 & 0 & 2\sqrt{R_s R_p} \end{pmatrix} \quad (9)$$

where $C_2 = \cos(2\theta)$ and $S_2 = \sin(2\theta)$.

For a metallic reflection, $\theta = p + 90^\circ$ (Borra 1976).

Hence, one finds that k , n , ϕ , and p fully describe a reflection.

Phase shifts:

The phase shifts of the p and s planes must be found for aluminium. Hass and Waylonis (1961), have found the optical constants (n and k) for aluminium for a variety of wavelengths. Table one summarizes the results.

Table 1

λ (nm)	300	340	380	436	492	546	578	650
n	.25	.31	.37	.47	.64	.82	.93	1.30
k	3.33	3.80	4.25	4.84	5.50	5.99	6.33	7.11

Based on MMT technical reports, Figure one shows the optical schematic for one primary mirror. The angle of incidences for the tertiary (ϕ_t) and the beam combiner (ϕ_{bc}) are noted.

$$\phi_t = 40.809$$

$$\phi_{bc} = 42.063$$

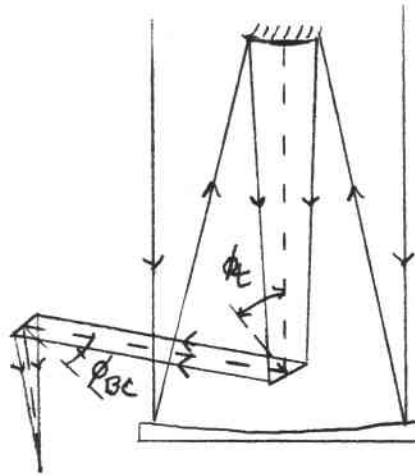


Figure 1

Using equations 1,3,and 4, the following graph shows the phase shifts of the tertiary and beam combiner. The shifts are expressed as fractions of a wavelength. Because the angle of incidences of the two mirrors differ, the phase shifts are not the same. However, they seem to converge for longer wavelengths, and probably asymptotically approach one-half wave for the infrared.

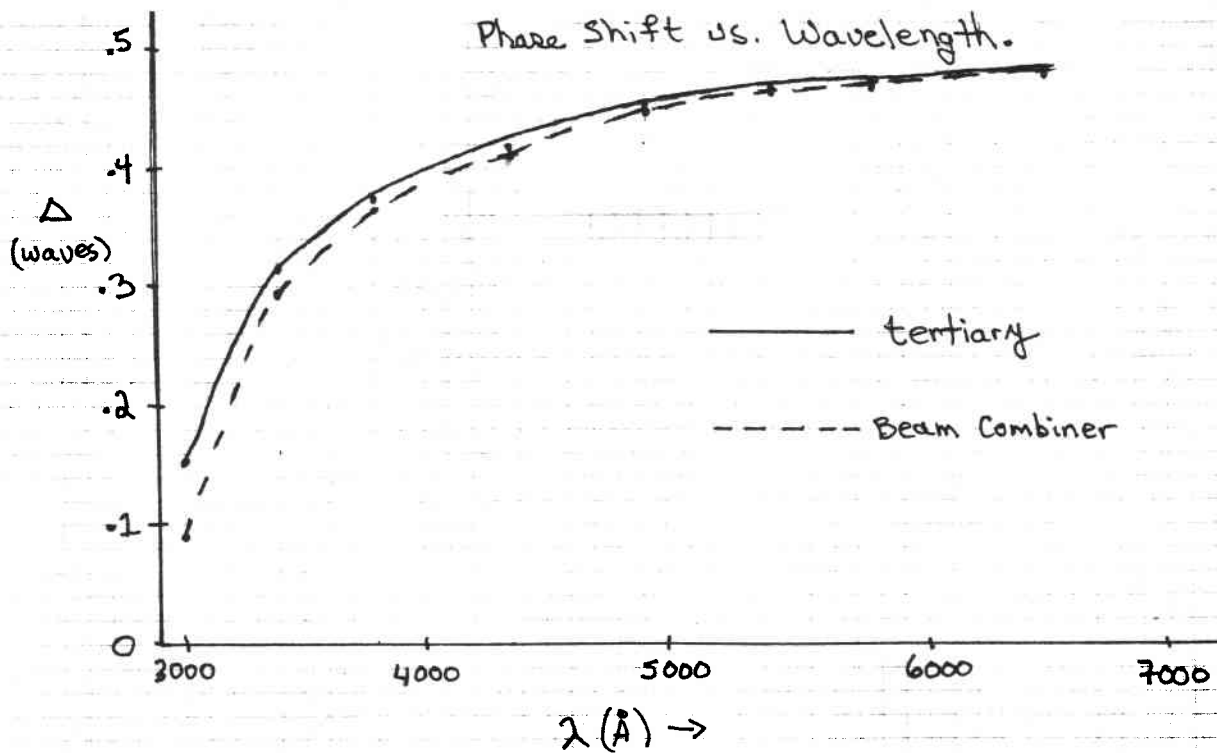


Fig. 2

Table 2 summarizes the data (as these are useful numbers):

Table 2

λ (nm)	tertiary		beam combiner	
	(waves)	(rads)	(waves)	(rads)
300	.16	.992	.09	.561
340	.32	2.00	.30	1.87
380	.38	2.37	.36	2.29
436	.42	2.63	.41	2.59
492	.45	2.82	.44	2.80
546	.46	2.91	.46	2.90
578	.47	2.95	.47	2.94
650	.48	3.02	.48	3.02

Care must be taken when performing the inverse tangent operation of equation 1.

Since the azimuths of the fast axes for both the tertiary and beam combiner are in the plane of incidence of figure 1, all of the factors needed to calculate the Mueller matrices are now known.

Retarder vs. retarder + polarizer:

At this point, I wish to compare the results of several incident polarizations injected into one primary mirror train with respect to treating each mirror as a retarder only, and then each mirror as a retarder plus polarizer.

Treating mirrors as retarders only is a good first approximation, and it will be interesting to find the relative order of magnitude of the contributions from the polarizer.

The instrumental effects will only be considered for the tertiary and beam combiner as the reflections from the primary and secondary mirrors are nearly normal and cancel each other (ie. each imparts a 1/2 wave phase shift, therefore both give a full wave).

To make the results more meaningful, I shall introduce a dielectric retarder after the beam combiner that cancels the effects of the retardance of the two mirrors. (It will be perpendicular to the optical axis). From figure 2, it is seen that the phase shift of each mirror at 492 nm is 2.82 and 2.799 radians. If the total retardance is 2π radians, the optical train will have a retardance of 1 wave = no effect. Therefore, the retardance of this simple compensator is $2\pi - 2.82 - 2.799 = 0.661$ radians. Its azimuth should be aligned with the azimuths of the beam combiner and tertiary.

I programmed my computer to do matrix multiplication of Mueller matrices and Stokes vectors (Prgm listing occurs at end of paper).

Table three lists the results of several incident polarization states injected into the optical path of one primary mirror. Column one is the Stokes vector in.

Column two is the vector out if the mirrors act only as retarders (plus the compensator).

Column three is the vector out if the mirrors are considered as partial polarizers (plus the compensator).

Table 3

In	descript.	out (R)	out (PP)
(100, 0, 0, 0)	Unpolarized	(100, 0, 0, 0)	(100, 1.9, 3.9, 0)
(100, 100, 0, 0)	linear	(100, 100, 0, 0)	(100, 99.9, 3.9, .27)
(100, $\frac{100}{\sqrt{2}}$, $\frac{100}{\sqrt{2}}$, 0)	linear	(100, $\frac{100}{\sqrt{2}}$, $\frac{100}{\sqrt{2}}$, 0)	(100, 69.7, 71.7, .09)
(100, 0, 0, 100)	RCP	(100, 0, 0, 100)	(100, 1.6, 4.1, 99.9)
(100, $\frac{100}{\sqrt{3}}$, $\frac{100}{\sqrt{3}}$, $\frac{100}{\sqrt{3}}$)	ellipt	(100, $\frac{100}{\sqrt{3}}$, $\frac{100}{\sqrt{3}}$, $\frac{100}{\sqrt{3}}$)	(100, 57.5, 59.7, 55.9)

In the Mueller representation, the input intensity (I) is