

Technical Report 27

**COMPARISON OF SECONDARY SUBSTRATE
MATERIAL AND CONSTRUCTION**

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There are several requirements that must be met in the selection of a substrate for secondary mirrors. First, the substrate must be rigid enough to hold its shape under the stress of fabrication (lapping), and to resist gravitational flexure within a given design error budget. Second, it must resist deformations due to thermal transients in the telescope environment (again within the error budget). Third, taken together with its mount, the entire secondary package should achieve isothermality with the ambient air in about the same time as an actively cooled borosilicate honeycomb primary--namely, one hour. Finally, the chosen substrate must meet these requirements at a reasonable cost.

A survey of commercially available substrate materials and construction techniques leads to a bewildering range of choices and costs. Some simple scaling techniques can be used to compare relative flexure and thermal response. These techniques are not accurate enough to provide detailed design of a substrate and mount, however they can be used to compare the relative performance of such diverse substrates as, say, a ULE meniscus and an aluminum foam-cored sandwich.

NOTATION

The notation used is similar to Mehta's¹:

b = Rib spacing
t_f = Front facesheet thickness
t_b = Equivalent bending thickness
t_w = Web thickness
A = Area of mirror surface
D = Mirror diameter
E = Young's modulus
F = Flexural rigidity
H_c = Core depth or rib height
I = Sectional moment of inertia
W = Mirror weight
η = Core or rib solidity ratio
ρ = Density
ν = Poisson's ratio

The following are also used:

c = Material specific heat
h = Heat transfer coefficient
k = Material thermal conductivity
v = Wind velocity
A_r = Area of rib walls
C = Thermal capacitance

L = Characteristic length
M = Mass
P_p = Polishing pressure
Q_{rms} = RMS surface error due to "quilting" during polishing
R = Thermal resistance
R_o = Kolmogorov's atmospheric scaling parameter
T_a = Ambient air temperature
T_c = Secondary cell temperature
T_m = Secondary mirror temperature
α = Coefficient of linear thermal expansion
ε = Surface emissivity
τ = Thermal time constant

CAVEATS

The equations used strictly apply only to flat plates or symmetrically curved shapes - biconcave or biconvex. Shear effects are neglected here, but should not be overlooked in any final design. Only axial support systems are considered since these are generally more complex than lateral support systems. Certain assumptions about the structure of lightweight sandwich and ribbed substrates are explained later. The comparisons should be considered only as relative and approximate.

BASIS OF COMPARISON

Rather than develop predictions of the absolute gravity flexure on arbitrary support systems, a single mirror and support system, the ESO NTT, was used as a basis of comparison. The NTT primary has demonstrated flexure commensurate with R_o = 160 cm or a FWHM of about .06 arcseconds.² All comparisons between substrates are made for absolute gravity-induced flexure equal to that of the NTT. For secondary mirrors, R_o must be scaled by the magnification of the wavefront onto the secondary--i.e. the ratio of primary to secondary diameters. For example, the f/5.4 secondary is about 1/4 the primary diameter, hence the NTT equivalent support system scales to R_o = 4 x 160 or about 540 cm equivalent free atmosphere.

The support system for the NTT was modeled on a 1 meter mirror and successfully scaled to a 3.5 meter mirror.³ It consists of 78 axial support points grouped in four concentric rings of 8, 16, 24 and 30 points. The mirror and support system can be conveniently truncated to develop estimates of substrate performance on 78, 48 or 24 support points.

SCALING LAWS

The NTT successfully demonstrated the well known scaling law for constant gravitational flexure:

$$(1) \quad \frac{t_f^2}{D^4} = \text{constant}$$

The application of this scaling law is limited to solid mirrors of the same material. In order to compare menisci of different materials, the flexural rigidity (F), which is material dependent, must remain constant:

$$(2) \quad F = \frac{E t_f^3}{12 (1 - \nu^2)} = \text{constant}$$

Equations 1 and 2 can be used to compare thickness and weight of menisci of various sizes and materials. To compare ribbed and honeycomb sandwich structures it is necessary to find an equivalent flexural rigidity. To do this, F is redefined using t_b , the equivalent bending thickness,

$$(3) \quad F = \frac{E t_b^3}{12 (1 - \nu^2)}$$

and t_b can be found from

$$(4) \quad t_b^3 = \frac{(1 - \eta/2) (t_f^4 - \eta H_c^4/2) + (t_f + H_c)^4 \eta/2}{(t_f + \eta H_c/2)}$$

for ribbed structures or

$$(5) \quad t_b^3 = (2t_f + H_c)^3 - (1 - \eta/2) H_c^3$$

for honeycomb sandwich construction. This assumes the back and front facesheets are of equal thickness.

Given a mirror diameter, the order of calculation for comparing a meniscus to a sandwich or ribbed structure runs as follows: First, find the thickness of the Zerodur meniscus scaled from the NTT on either 78, 48 or 24 support points using equation 1. Next convert to a new material and thickness using equation 2. Then, iteratively solve equation 4 or 5 to find the equivalent core or rib height.

The weights of the various mirrors can be found from:

$$(6a) \quad W = A \rho t_f \quad (\text{Meniscus})$$

$$(6b) \quad W = A \rho (t_f + H_c \eta) \quad (\text{Ribbed})$$

$$(6c) \quad W = A \rho (2t_f + H_c \eta) \quad (\text{Sandwich})$$

The surface area of the rib walls, A_r , is given by:

$$(7) \quad A_r = \frac{4A H_c}{b - t_w}$$

which holds for triangular, square or hexagonal cellular core construction.

HONEYCOMB SANDWICH AND RIBBED STRUCTURES

The calculation outlined above will not converge unless certain parameters are set ahead of time. These are the facesheet thickness (t_f), and the core or rib solidity ratio (η). η is in turn set from the *a priori* choice of rib spacing (b) and rib thickness (t_w):

$$(8) \quad \eta = \frac{(2b + t_w) t_w}{(b + t_w)^2}$$

In his paper Mehta indicates that lower values of η always yield higher stiffness-to-weight ratios. This conclusion may not hold when shear effects are taken into account. Despite this,

for the secondary mirrors under consideration there is considerable improvement in weight, rigidity and thermal response by using rib spacings of about 10 cm instead of the 20 cm spacing presently used for casting primaries. For this comparison 5 mm thick ribs on 10 cm centers have been assumed for both sandwich and ribbed construction. This is consistent with most (but not all) commercially available construction.

The facesheet should be as thin as possible consistent with the limitation imposed by surface quilting during polishing. English and Takke⁴ give an expression for the RMS surface error due to quilting as:

$$(9) \quad Q_{rms} = \frac{C P_p (b - t_w)^4 (1 - \nu^2)}{E t_f^3}$$

where C is a constant empirically determined as $C = .0045$ for Q_{rms} in waves at .6 microns. Comparison to experience at SOML with the stress lap indicates a slightly more conservative value of $C = .005$ is appropriate and has been adopted for this comparison.

Table 1 lists the face sheet thickness for the various materials calculated from equation (9) and adopting a 10 cm rib spacing. These thicknesses form the basis for subsequent calculations in equations 4 and 5.

Table 1: Facesheet thickness for various materials for 6 nm RMS quilting with .1 psi lap pressure. Ribs are 5 mm thick on 10 cm centers.

Pyrex	9.0 mm
Quartz	8.5
ULE	8.8
Zerodur	8.0
Aluminum	7.1
Beryllium	5.4
SiC	4.5

MATERIALS

Seven materials have been carried in this comparison. These are listed along with their various characteristics in Table 2. In addition to the basic material properties, derived parameters are also listed.

It can be seen that Pyrex is more susceptible to thermal bending than the other materials. It is also slower to respond to a temperature change (low thermal diffusivity). Though this does not eliminate Pyrex as a candidate material, it does indicate that active thermal control will probably be necessary for Pyrex substrates.

Not all the materials listed are available in the forms of construction considered. Some, such as beryllium and aluminum, are limited by current polishing technology and are included as possible candidates only for the infrared (f/15) secondary.

Table 2: Material Properties

	ρ	E	k	c	ν	α			
	10^3 Kg/m^3	10^{10} N/m^2	W/m/K	J/Kg/K	--	ppm/K	Ek/α	$k/\rho/c$	E/ρ
Pyrex	2.23	6.17	1.02	835	.200	3.20	1.9	0.55	27.6
Quartz	2.20	7.32	1.37	721	.167	.56	17.9	0.84	33.2
ULE	2.20	6.77	1.31	766	.176	.03	295.6	0.78	30.7
Zerodur	2.53	9.06	1.65	812	.120	.12	124.5	0.80	35.8
Aluminum	2.71	11.70	171.00	960	.334	23.90	83.9	66.00	43.1
Beryllium	1.85	30.40	220.00	1820	.025	11.20	597.1	65.00	164.3
SiC	3.10	46.60	200.00	700	.280	2.40	3883.3	89.00	150.3

Ek/α is a measure of resistance to bending due to thermal transients. $k/\rho/c$ is also called thermal diffusivity. It is inversely proportional to the time to come to thermal equilibrium. E/ρ is also called specific stiffness. It is an index of resistance to self-weight-induced flexure. For each of these derived parameters high values are desirable.

EQUILIBRIUM TEMPERATURE

Secondary packages typically have a total surface area about four times the mirror surface, with about one quarter pointed directly at the sky. Beckers and Williams⁵ measured the temperature of steel beams coated with different materials exposed to the night sky and derived a thermal equilibrium equation in which radiative cooling to the night sky is balanced by convective warming from the ambient air. Based on their findings, the equilibrium temperature for a secondary package is:

$$(10) \quad T_c = \frac{T_a - 50\epsilon}{9 + \nu}$$