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Approximate Wind Disturbance of the MMT 6.5-m Primary Mirror on its Supports

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The wind-driven vibration spectrum of the MMT 6.5-m mirror on its support system is estimated. The mirror cell is considered attached to an infinitely rigid telescope mount and the mirror itself is considered rigid (*i.e.*, the response of the telescope mount, drive gearing, and mirror bending modes are neglected). The six hardpoints (stiff springs) couple the mirror to the cell. The distributed air supports and servo inner-loop provide damping, error correction, and forces to support the weight of the mirror. The results obtained here will be used to help place lower limits on the required hardpoint stiffness.

The power spectrum of the longitudinal component of windspeed (component parallel to mean wind) is used to calculate a vibration response of the primary mirror on its supports. Spatial filters are applied to the transverse wind spectra in order to approximate turbulence decorrelation and to estimate moments across the mirror arising from pressure gradients. The rms responses are compared to misalignment errors that produce 0.1 arcsec rms images. The relationship of the vibration response to the rms atmospheric pathlength error is briefly discussed. A lower limit for hardpoint stiffness will be suggested so that the rms primary mirror response is smaller than these errors.

Two hardpoint stiffnesses are considered (40 and 80 N/μm) and two mean wind velocities (6.7 and 22 m/s) with three damping cases for each. We assume that the telescope has an elevation of 30 degrees at an azimuth pointed directly into the mean wind vector. No attenuation of the wind by the telescope enclosure is considered (When the telescope is pointed into the wind, the enclosure design -- combined with the rear building modification -- should produce little attenuation of the wind). The calculations are carried out in two parts:

1) Part 1 uses a one-dimensional longitudinal wind power spectral density (PSD) to analyze the axial and lateral vibrations of the primary mirror. Since the 3-D PSD is not used, the wind is considered to have the same instantaneous force everywhere on the mirror (i.e. no wind decorrelation across the exposed area is considered). Tilt/moment excitations are neglected. Finally, the theoretical wind spectrum used here is compared to 1-D wind PSDs measured on Mt. Hopkins and those used for the VLT.

For a single aperture, axial vibrations do not affect image quality unless they are large enough to defocus the telescope. They clearly affect the use of the telescope as an element of an interferometer. Lateral vibrations cause tilt of the wavefront and therefore image motion.

2) Part 2 incorporates the lateral variations of the longitudinal component of wind in order to estimate the modifications to the results of Part I produced by more realistic wind. This allows revised calculations for piston and lateral responses that include the effects of wind decorrelation across the exposed area. In addition, tilts and moments arising from uneven pressure distributions are

approximated.

Part 3 collects the results and compares them to the telescope optical specifications and to the rms pathlength errors the atmosphere produces.

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Constants and definitions:

$r2d := \frac{180}{\pi}$ Rad to Deg conversion.

A	Area of mirror m^2.
C_D	Drag coefficient.
D	Mean Drag force N.
f	Dimensionless frequency.
H, H_{ho}, H_s	Total, harmonic oscillator, and servo force admittances.
M_p	Mass of the primary mirror kg.
n, n_r	Frequency and resonant frequency Hz.
PSD	Power spectral density.
S, S_{hp}	Hardpoint stiffness N/m.
S_D	Drag spectral density N^2/Hz.
S_u	Longitudinal turbulent wind velocity PSD $(m/s)^2$/Hz.
S_{uu}	Longitudinal PSD of longitudinal wind fluctuations $(m/s)^2$/unitξ.
S_{uv}	Lateral PSD of longitudinal wind fluctuations $(m/s)^2$/unitη.
S_{uw}	Vertical PSD of longitudinal wind fluctuations $(m/s)^2$/unitζ.
S_v	Lateral wind fluctuation PSD along the wind vector.
S_x	Vibration displacement PSD m^2/Hz.
u_f	Friction or shear velocity m/s.
$U = V_z$	Mean wind velocity at height z m/s.
u, v, w	Longitudinal, lateral, and vertical wind fluctuations.
X	RMS vibration displacement m.
x, y, z	Directions along, lateral, and vertical to the wind vector.
z	Height above ground (m) and vertical transverse direction.
z_o	Ground roughness constant m.
γ	Damping constant (ratio to critical damping).
η	Spatial frequency tranverse horizontal to the wind (cycles/m).
ρ	Air density kg/m^3.
ω_{op}, ω_{ol}	Piston and lateral support resonances rad/s.
ξ	Spatial frequency along the wind vector (cycles/m).

ζ Spatial frequency transverse vertical to the wind (cycles/m).

Part I: Mirror Response to a 1-D Wind Power Spectrum

I 1. Hardpoint Stiffness and Resonant Frequencies:

SGH FEA Models

Simpson, Gumpertz, and Heger SGH (Antebi 1994) calculated the resonant frequencies for the 6.5-m mirror (attached to the cell and trunnion ring) for a hardpoint stiffness of S = 40 N/μm:

*	Lateral translation	11.2 Hz
*	Rotation about optic axis	13.3 Hz
*	Rotation about diameter	20.7 Hz
*	Piston translation	21.4 Hz

Miglietta (1994) has calculated modal stiffnesses for the mirror and supports attached to an infinitely rigid cell (not used here).

Scaling the lateral and axial resonances from Antebi:

$$\omega_{op}(S) := 2\cdot\pi\cdot 21.4\cdot\sqrt{\frac{S}{40\cdot 10^{6}}}$$

Piston resonance (rad/s) with hardpoint stiffness S in N/m.

$$\omega_{ol}(S) := 2\cdot\pi\cdot 11.2\cdot\sqrt{\frac{S}{40\cdot 10^{6}}}$$

Lateral resonance.

Summary of resonances used for this work.

	S=40 N/μm	S=80 N/μm
Piston (Hz)	21	30
Lateral (Hz)	11	16

Simple Check

We can perform a simple check of these resonances by making a geometrical calculation of the mirror attached to an infinitely rigid cell:

$M_p := 8435$ Mass of mirror (kg)-- (Hill 1994a).

$$\omega_{cp}(S) := \sqrt{\frac{6 \cdot S \cdot \cos\left(\frac{57}{r2d}\right)}{M_p}}$$

Approximate piston resonant frequency of mirror (rad/s). S is the hardpoint stiffness along its axis (N/m). The cos term results from the angle of the harpoint attachment from the cell to the mirror backplate.

$$\omega_{cl}(S) := \sqrt{\frac{4 \cdot S \cdot \sin\left(\frac{57}{r2d}\right) \cdot \cos\left(\frac{60}{r2d}\right)}{M_p}}$$

Approximate lateral resonance.

$S_{hp} := 80 \cdot 10^6$ Test hardpoint stiffness (N/m).

$\nu_{cp} := \frac{\omega_{cp}(S_{hp})}{2 \cdot \pi}$ $\nu_{cp} = 28$ Piston Resonant frequency of mirror (Hz).

$\nu_{cl} := \frac{\omega_{cl}(S_{hp})}{2 \cdot \pi}$ $\nu_{cl} = 20$ Lateral resonance (Hz).

These simple approximations compare fairly well to the 30 and 16 Hz resonances from SGH implying that the SGH cell does not degrade the hardpoint stiffnesses.

I 2. Dynamic wind forcing functions

To estimate the rms amplitude of the wind-induced displacements of the primary mirror, the Kaimal expression for the dynamic wind PSD is used (see, e.g., Kaimal et al.1972 and Simiu 1974). Much of the astronomical literature, however, (e.g. Castro et al. 1994 and Forbes et al.1982) uses the Davenport spectrum while the wind tunnel tests of the VLT (Ravensbergen 1994) use the von Karman spectrum. Although each of these spectra share a -5/3 Kolmogorov frequency dependence for high frequencies (n > 1Hz), the Kaimal spectrum represents an improved PSD vs. frequency by providing a better description of low frequency turbulence and the proper description of PSD vs. height above the ground (see Simiu & Scanlan 1986--chapter 2 for a general overview).

For frequencies greater than 0.1 Hz, the turbulent wind power spectrum has a universal description that depends only on the roughness of the surface over which the wind flows, the mean velocity, and the height. It is independent of weather conditions (atmospheric temperature stability). For low frequencies (n<0.1 Hz) however, the power spectra are strongly dependent on the temperature stability of the atmosphere. In this region, an unstable atmosphere has orders of magnitude more energy than a highly stable weather pattern.

To simplify this work, the universal function is extended to low frequencies. In practical terms, this corresponds to an atmosphere that is of average stablity and so gives a believable estimate of typical low frequency energy spectra. One can be a bit sloppy in this region because the support servo inner-loop error correction is most effective at low frequencies.

In part I, only the longitudinal variations of wind along the x direction S_{ux} are considered. Because the transverse dependence of u fluctuations is neglected, the turbulence correlation across the surface of the mirror is unity. This means that all points on the mirror see the same instantaneous forces. In reality, the correlation is less than 1 and pitch and yaw moments will be imparted to the mirror thus taking energy out of the lateral or piston vibrations. Further complications (which are neglected in both parts I and II) occur as the wind passes over the enclosure, cell, and OSS.

The situation has been resolved empirically for radio telescopes by wind tunnel testing, and complete descriptions of moments and forces exist (Fox 1962a,b, Blaylock 1964, and Bicknell 1962). The applicability of these results to an optical telescope geometry would require careful consideration.

The aerodynamic admittance function provides a convenient modification of the drag coefficient so that the 1-D PSD includes an estimate of the blocking and decorrelation of dynamic wind force over the exposed telescope area. For the VLT (Cullum, 1994; Ravensbergen 1994), the aerodynamic admittance was measured with a model telescope placed into a wind tunnel. The model had the louvers and bottom doors closed, the wind screen adjusted to provide 20% permeability, the telescope pointed at zenith, and the axes locked. (This appears to be the most enclosed geometry possible with the telescope oriented to maximize sensitivity to the wind.)

Because similar data are unavailable for the MMT, we must approximate the response of the mirror to the wind by using wind power spectra available from the literature. Since some power spectra have been measured at the MMT, we have a partial "sanity" check of our results.

I 2a. Longitudinal velocity fluctuations u along the direction of wind flow x.

$z := 10$ Height above ground (m).

$z_o := 0.08$ Roughness constant. Chosen here for sparsely built-up open terrain.

$k := 0.4$ Empirically determined constant.

$$f(n, V_z) := \frac{n \cdot z}{V_z}$$

Dimensionless frequency (n is frequency in Hz and V_z is the mean longitudinal wind velocity at height z (m/s).

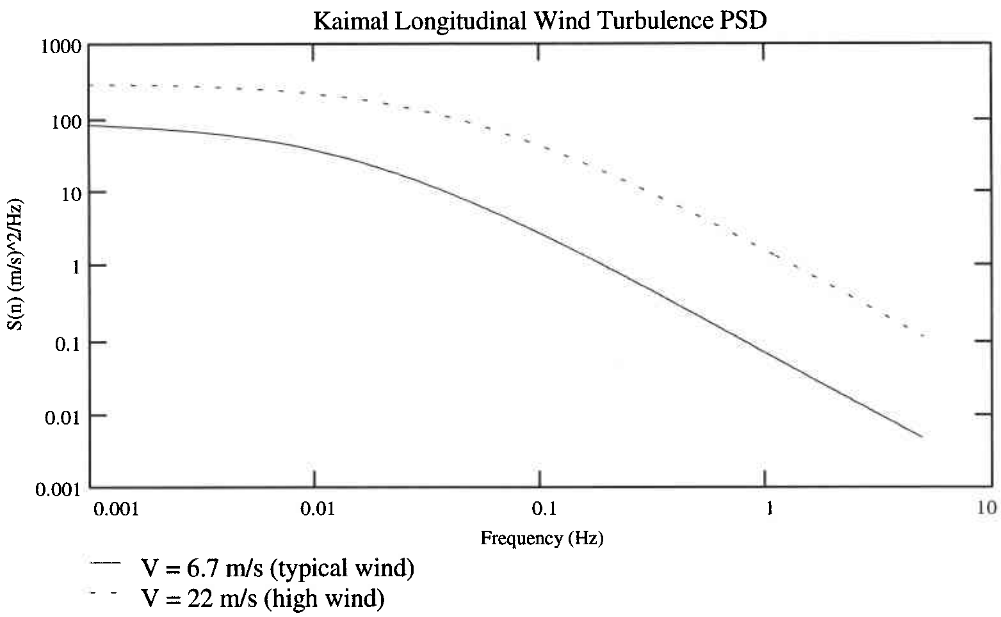
$$u_f(V_z) := \frac{V_z \cdot k}{\ln\left(\frac{z}{z_o}\right)} \qquad u_f(10) = 0.83$$

Friction velocity (m/s).

$$S_{ux}(n, V_z) := \frac{200 \cdot f(n, V_z)}{\left(1 + 50 \cdot f(n, V_z)\right)^{\frac{5}{3}}} \cdot \left(\frac{u_f(V_z)^2}{n}\right)$$

Kaimal et al. longitudinal energy spectrum $(m/s)^2/Hz$).

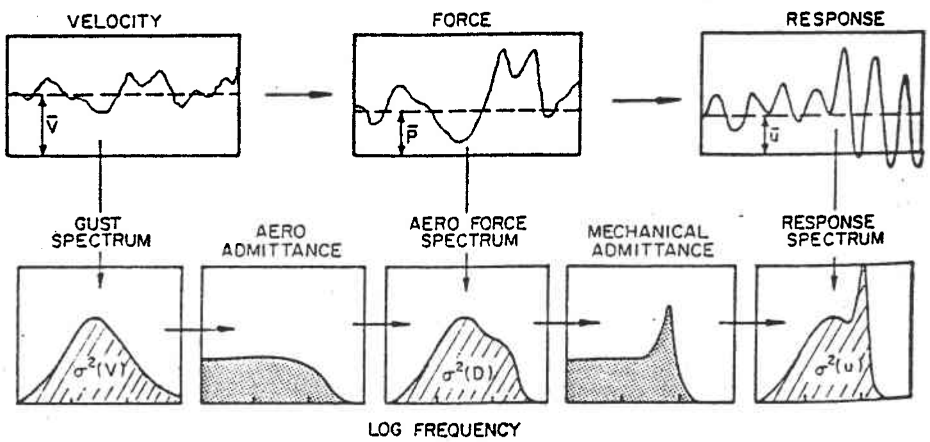
$n := .001, .002 .. 5$



I 3. Converting a turbulence PSD into a displacement PSD:

I 3a. Approach

The following diagram (Harris and Crede 1987--chpt 29) illustrates the process of converting a time-series turbulent wind velocity into a response power spectral density.



The gust PSD is given in the previous section. The conversion of the gust spectrum into a drag force spectrum requires a Taylor approximation of the time-dependent drag force. The process used in this report is as follows:

Time-dependent drag:

$$D(t) = 1/2\rho A C_D V^2(t),$$

but $V(t) = V_z + v(t)$, where $v(t)$ is the variation of the turbulent velocity. Expanding $(V_z + v(t))^2$ in a Taylor series gives: $V_z^2(1 + 2v(t)/V_z + v^2(t)/V_z^2 + ...)$. So that:

$$D(t) \sim D + 2\, D/V_z\ v(t))$$

where D is the static drag force at height z produced by the mean wind V_z. $v(t)$ transforms into $S_u(n)$ above, and the amplitude squares so that the drag spectrum becomes (see, e.g., Harris & Crede 1987--chpt 29):

$$S_D(n) \sim 4\,(D/V_z)^2\, S_u(n)$$