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Stewart Platform Matrices for the 6.5m MMT

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Abstract

The MathCad worksheets used to derive the coordinate transformations for various Stewart platforms for the 6.5m MMT are documented. Thus far, these worksheets have successfully provided matrix transformations for the primary mirror hardpoint system, and the f/9, f/15, and f/5 secondary mirror hexapods. At this time, all matrices except for the f/5 have been fully tested and found to work very well.

I. Intro

The 6.5m MMT contains Stewart platform[1] positioning systems for : 1) the primary mirror, 2) the f/9-f/15 secondaries shared positioner, and 3) the f/5 secondary mirror.

The approaches I've taken for developing these platform matrices have been previously summarized and explained as they relate to the primary mirror hardpoint system for coordinate transformations[2] and force-torque conversions[3]. The testing of the matrices for the f/9 hexapod have likewise been documented[4]. The f/15 tests were conducted with the AO group, but were undocumented.

Briefly, the coordinate transformations consist of defining the actuator axes intersections near the fixed and mobile plates (using the desired rotation point as the origin). The mobile intersection points are then rotated and translated with Euler matrices. The influence of these cartesian motions on the strut lengths define the matrix elements of the inverse kinematic solution. Appendix A gives a listing of the MathCad worksheet that provides this solution for the f/5 hexapod. All the calculations in this worksheet are identical for the other Stewart platforms with the exceptions of the actuator intersection coordinate files (*e.g*. hexupper and hexlower).

Appendix B lists the MathCad worksheet for determining the forces and torques on the Stewart platform as a function of cg and elevation. The example is for the f/9 hexapod with the f/9 secondary mirror system.

II. References

[1] D. Stewart, "A platform with six degrees of freedom," *Proc. Instr. Mech. Engr*. 180, 371-386 (1965).

[2] S. C. West, "A hardpoint calculator for the MMT conversion," *MMTO Conversion Technical Memo* #96-1 (March 1996).

[3] S. C. West, Hardpoint platform matrices for the MMT 6.5m primary mirror cell," *MMTO Conversion Internal Technical Memo* #**97-2** (1997).

[4] S. C. West, D. Fisher, and P. Spencer, T. Trebisky, B. Hille, and G. Weir, "6.5m MMT f/9-f/15 Hexapod Laboratory Calibration," *MMTO Conversion Technical Memo* #**00-3**, (07 Mar 2000)

Appendix A:

Strut length calculator for the f/5 MMT hexapod

This worksheet calculates the changes in the lengths of hexapod struts vs rigid body motions of mobile attachment plate.   The strut lengths are calculated as the distance between the upper and lower actuator intersection points. For motion of the hexapod, the mobile plate intersections are rotated and translated. The new mobile coordinates relative to the fixed coordinates give the relative strut lengths. The coordinates of the mobile attachment points are determined using the Euler matrix in xyz convention applied at an arbitrary rotation point (e.g. mirror vertex, secondary center of curvature [zero wavefront tilt with coma], or primary prime focus [zero coma with wavefront tilt]. The lower strut connections are considered fixed in space.

+X is east, +Y is north, and +Z is skyward away from the primary mirror vertex.

2-11-03: added math to change the rotation point in the worksheet rather than changing the hpupper/lower files. The rotation point is the *origin* of the actuator intersection coordinates, and this worksheet contrains it to be on the z-axis. This amounts to a simple z-translation of those coordinates.

f5strutLength.mcd; derived from f9strutLength.mcd.
scw: 2-19-96; update: 4-29-97, 2-11-03

Pivot point:

Distance of the effective rotation point relative to the mobile actuator intersection plane (mm). This distance is negative if the point is below this plane (i.e. toward the primary mirror), and positive if its toward the fixed hexapod plate.

$pp := 0$

vertex rotation pp = -209.78 mm (these are for bare Cass hex position)
PF rotation pp = 1733.32 mm
Rv rotation pp = 4941.11 mm

Strut Attachments:

The upper set of attachment points move with the mirror (mobile) while the lower set remain fixed.

$wpmobile := READPRN("f5hexupper")^T$

Read workpoint coordinates near the mobile platform and transpose into column vectors (mm) with 0,0,0 at the mirror vertex in the operating position.

$wpfixed := READPRN("f5hexlower")^T$

Read strut workpoint coordinates near the fixed platform.

$wpmobile := wpmobile - T(0,0,pp)$

$wpfixed := wpfixed - T(0,0,pp)$

Change the workpoint z-coords to correspond to the desired rotation point (0=center of the mobile actuator intersection plane). see above for sign conv

$wpvector := (wpfixed - wpmobile)$

Strut vector elements (mm) -- base of vector is at upper workpoint.

$i := 0..5$

$hplength_i := \left|wpvector^{\langle i\rangle}\right|$

$$hplength = \begin{pmatrix} 831.535 \\ 831.537 \\ 831.537 \\ 831.537 \\ 831.537 \\ 831.535 \end{pmatrix}$$

Nominal hardpoint lengths in mms. Errors of ~8 microns due to ACAD position uncertainties.

End coordinate modifications:

The above vectors are usually defined by the intersections of the strut axes. This section allows trimming their lengths and end coordinates to correspond with the actual flexure attachments. For forces, and torques, all that matter are properly oriented vectors, but for positioning, the locations of the flexures are important too (I think).

Projections of the strut vectors onto each axis:

$$xyz := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Axis unit vectors

$$wpdc := \left| \begin{array}{l} \text{for } j \in 0..2 \\ \quad \text{for } i \in 0..5 \\ \quad\quad dc_{j,i} \leftarrow \dfrac{wpvector^{\langle i\rangle} \cdot xyz^{\langle j\rangle}}{\left|wpvector^{\langle i\rangle}\right| \cdot \left|xyz^{\langle j\rangle}\right|} \\ dc \end{array} \right.$$

Calculate the direction cosines of the working point vectors using the dot product. In addition to projections onto the x,y,z axes, they serve as unit vectors for the hardpoints.

$$wpdc = \begin{pmatrix} -0.547 & -0.104 & 0.443 & -0.443 & 0.104 & 0.547 \\ 0.195 & -0.571 & 0.376 & 0.376 & -0.571 & 0.195 \\ 0.814 & 0.814 & 0.814 & 0.814 & 0.814 & 0.814 \end{pmatrix}$$

Direction cosine vectors for the struts. (The base of the vector is at the output plane working point.)

$$\overrightarrow{acos(wpdc)\cdot\frac{180}{\pi}} = \begin{pmatrix} 123.146 & 95.978 & 63.728 & 116.272 & 84.022 & 56.854 \\ 78.731 & 124.836 & 67.926 & 67.926 & 124.836 & 78.731 \\ 35.496 & 35.496 & 35.496 & 35.496 & 35.496 & 35.496 \end{pmatrix}$$

Vectors giving the angles that the struts make with the x,y,z axes (degrees).

$$wpmobile = \begin{pmatrix} 454.66 & 454.66 & 0 & 0 & -454.66 & -454.66 \\ 262.5 & 262.5 & -525 & -525 & 262.5 & 262.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Raw workpoint coordinates.

$$wpfixed = \begin{pmatrix} 0 & 368.061 & 368.061 & -368.061 & -368.061 & 0 \\ 425 & -212.5 & -212.5 & -212.5 & -212.5 & 425 \\ 677 & 677 & 677 & 677 & 677 & 677 \end{pmatrix}$$

$trim := 0$

amount to trim off the ends of the wp vectors to reach the flexures. Really needs seperate upper and lower trim dimensions...

$i := 0 .. 5$

$$hpmobile^{\langle i\rangle} := wpmobile^{\langle i\rangle} + trim\cdot wpdc^{\langle i\rangle}$$

$$hpfixed^{\langle i\rangle} := wpfixed^{\langle i\rangle} - trim\cdot wpdc^{\langle i\rangle}$$

$$hpmobile = \begin{pmatrix} 454.66 & 454.66 & 0 & 0 & -454.66 & -454.66 \\ 262.5 & 262.5 & -525 & -525 & 262.5 & 262.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

New strut vector coordinates having endpoints at the flexures.

$$hpfixed = \begin{pmatrix} 0 & 368.061 & 368.061 & -368.061 & -368.061 & 0 \\ 425 & -212.5 & -212.5 & -212.5 & -212.5 & 425 \\ 677 & 677 & 677 & 677 & 677 & 677 \end{pmatrix}$$

$hpvector := (hpfixed - hpmobile)$

Form new difference vectors

$$\text{hplength}_i := \left|\text{hpvector}^{\langle i\rangle}\right| \qquad \text{hplength} = \begin{pmatrix} 831.535 \\ 831.537 \\ 831.537 \\ 831.537 \\ 831.537 \\ 831.535 \end{pmatrix}$$

Nominal hardpoint lengths in mms. Flexure-to-flexure.

Motion of the upper attachment points:

Input Fields:

$$\text{RAD1} := \frac{180}{\pi}$$

$\Delta x := 0.5 \qquad \Delta y := 0 \qquad \Delta z := 0$

Vertex translations.

$$\Delta\phi := \frac{0}{\text{RAD1}} \qquad \Delta\theta := \frac{0}{\text{RAD1}} \qquad \Delta\psi := \frac{0}{\text{RAD1}}$$

Rotations about z, intermediate y, and final x axes.

Euler matrix (in the xyz convention):

$$E(\phi,\theta,\psi) := \begin{pmatrix} \cos(\theta)\cdot\cos(\phi) & \cos(\theta)\cdot\sin(\phi) & -\sin(\theta) \\ \sin(\psi)\cdot\sin(\theta)\cdot\cos(\phi) - \cos(\psi)\cdot\sin(\phi) & \sin(\psi)\cdot\sin(\theta)\cdot\sin(\phi) + \cos(\psi)\cdot\cos(\phi) & \cos(\theta)\cdot\sin(\psi) \\ \cos(\psi)\cdot\sin(\theta)\cdot\cos(\phi) + \sin(\psi)\cdot\sin(\phi) & \cos(\psi)\cdot\sin(\theta)\cdot\sin(\phi) - \sin(\psi)\cdot\cos(\phi) & \cos(\theta)\cdot\cos(\psi) \end{pmatrix}$$

yaw (ϕ) about z, pitch (θ) about intermediate y, and roll or bank (ψ) about final x'.

$$E\left(0,0,\frac{0.05}{57.3}\right)\cdot\text{hpmobile}^{\langle 0\rangle} = \begin{pmatrix} 454.66 \\ 262.5 \\ -0.229 \end{pmatrix}$$

Translation matrix:

$$T(dx,dy,dz) \equiv \begin{pmatrix} dx & dx & dx & dx & dx & dx \\ dy & dy & dy & dy & dy & dy \\ dz & dz & dz & dz & dz & dz \end{pmatrix}$$

New coordinates of upper attachments:

$$newmobile := \overrightarrow{E(\Delta\phi, \Delta\theta, \Delta\psi) \cdot hpmobile - T(\Delta x, \Delta y, \Delta z)}$$

$$newmobile = \begin{pmatrix} 454.16 & 454.16 & -0.5 & -0.5 & -455.16 & -455.16 \\ 262.5 & 262.5 & -525 & -525 & 262.5 & 262.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$hpmobile = \begin{pmatrix} 454.66 & 454.66 & 0 & 0 & -454.66 & -454.66 \\ 262.5 & 262.5 & -525 & -525 & 262.5 & 262.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$newvector := hpfixed - newmobile$$

$$i := 0..5$$

$$newlength_i := \left| newvector^{\langle i \rangle} \right|$$

$$newlength = \begin{pmatrix} 831.262 \\ 831.485 \\ 831.759 \\ 831.316 \\ 831.589 \\ 831.809 \end{pmatrix}$$

Influence matrix of motions on hardpoint lengths (Inverse kinematic solution):

Calculate the change in strut lengths for very small motions of the mobile intersection points.

Hardpoint vector arrays for various motions of the mirror.

$$mobiledx := -\left(E(0,0,0) \cdot \overrightarrow{hpmobile - T(.1,0,0)} - hpfixed\right)$$

$$mobiledy := -\left[\left(\overrightarrow{E(0,0,0) \cdot hpmobile - T(0,0.1,0)}\right) - hpfixed\right]$$

$$mobiledz := -\left[\left(\overrightarrow{E(0,0,0) \cdot hpmobile - T(0,0,0.1)}\right) - hpfixed\right]$$

$$mobiled\phi := -\left[\left(\overrightarrow{E\left(\frac{0.05}{RAD1}, 0, 0\right) \cdot hpmobile - T(0,0,0)}\right) - hpfixed\right]$$

$$mobiled\theta := -\left[\left(\overrightarrow{E\left(0, \frac{0.05}{RAD1}, 0\right) \cdot hpmobile - T(0,0,0)}\right) - hpfixed\right]$$

$$mobiled\psi := -\left[\left(\overrightarrow{E\left(0, 0, \frac{0.05}{RAD1}\right) \cdot hpmobile - T(0,0,0)}\right) - hpfixed\right]$$

Now find the new lengths of the hardpoints for each motion of the mirror:

$i := 0..5$

$dxlength_i := \left|mobiledx^{\langle i\rangle}\right|$ $\qquad d\phi length_i := \left|mobiled\phi^{\langle i\rangle}\right|$

$dylength_i := \left|mobiledy^{\langle i\rangle}\right|$ $\qquad d\theta length_i := \left|mobiled\theta^{\langle i\rangle}\right|$

$dzlength_i := \left|mobiledz^{\langle i\rangle}\right|$ $\qquad d\psi length_i := \left|mobiled\psi^{\langle i\rangle}\right|$

Determine the influence of each motion on the differential length of the hardpoints:

$$inf^{\langle 0\rangle} := 1000\cdot\frac{hplength - dxlength}{.1} \qquad inf^{\langle 3\rangle} := 1000\cdot\frac{hplength - d\phi length}{.05\cdot 3600}$$

$$inf^{\langle 1\rangle} := 1000\cdot\frac{hplength - dylength}{.1} \qquad inf^{\langle 4\rangle} := 1000\cdot\frac{hplength - d\theta length}{.05\cdot 3600}$$

$$inf^{\langle 2\rangle} := 1000\cdot\frac{hplength - dzlength}{.1} \qquad inf^{\langle 5\rangle} := 1000\cdot\frac{hplength - d\psi length}{.05\cdot 3600}$$

$$inf^{\langle 0\rangle} := \frac{inf^{\langle 0\rangle}}{1000} \qquad inf^{\langle 1\rangle} := \frac{inf^{\langle 1\rangle}}{1000} \qquad inf^{\langle 2\rangle} := \frac{inf^{\langle 2\rangle}}{1000}$$

convert to um

$$inf = \begin{pmatrix} 0.547 & -0.195 & -0.814 & -1.127 & 1.795 & -1.036 \\ 0.104 & 0.571 & -0.814 & 1.126 & 1.795 & -1.036 \\ -0.443 & -0.376 & -0.814 & -1.127 & 0.000 & 2.072 \\ 0.443 & -0.376 & -0.814 & 1.126 & 0.000 & 2.072 \\ -0.104 & 0.571 & -0.814 & -1.127 & -1.795 & -1.036 \\ -0.547 & -0.195 & -0.814 & 1.126 & -1.794 & -1.036 \end{pmatrix}$$

Multiply this matrix by a column vector of dx,dy,dz,dϕ,dθ,dψ to get the change in hardpoint lengths in microns. 1st 3 cols have units of μm/mm and last three have μm/arcsecond.

WRITEPRN("f5matrix.dat") := inf

Final hardpoint lengths:

$$\text{motion} := \begin{pmatrix} 1000 \\ -1000 \\ 2000 \\ 300 \\ -500 \\ 200 \end{pmatrix}$$

um dx
um dy
um dz
arcsecond dϕ
arcsecond dθ
arcsecond dψ

$$\text{motion}_0 := \begin{pmatrix} 0 \\ 0 \\ -7000 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Length at rest on static supports.

$$\text{length} := \text{hplength} - \frac{\text{inf} \cdot \text{motion}}{1000}$$

$$\text{inf} \cdot \text{motion} = \begin{pmatrix} -2328.905 \\ -2861.958 \\ -1618.705 \\ -57.486 \\ -1951.602 \\ -951.881 \end{pmatrix}$$

$$\text{ength}_0 := \text{hplength} - \frac{\text{inf} \cdot \text{motion}_0}{1000}$$

$$\text{length} = \begin{pmatrix} 833.864 \\ 834.399 \\ 833.156 \\ 831.595 \\ 833.488 \\ 832.487 \end{pmatrix} \qquad \text{length} - \text{length}_0 = \begin{pmatrix} 8.028 \\ 8.561 \\ 7.318 \\ 5.757 \\ 7.651 \\ 6.651 \end{pmatrix} \qquad \text{struts} := \text{inf} \cdot \text{motion}$$

Influence of hardpoint length on mirror motions (partial direct kinematic solution):

$$\text{dl} := \begin{pmatrix} 20000 \\ 20000 \\ 20000 \\ 20000 \\ 20000 \\ 20000 \end{pmatrix}$$

Differential motions of struts (um).

First 3 rows have units of um/um and the last three have arcs/um.

$$inf^{-1} = \begin{pmatrix} 0.377 & -0.377 & -0.753 & 0.753 & 0.377 & -0.377 \\ -0.652 & 0.652 & -3.048 \times 10^{-5} & -1.481 \times 10^{-4} & 0.652 & -0.652 \\ -0.205 & -0.205 & -0.205 & -0.205 & -0.205 & -0.205 \\ -0.148 & 0.148 & -0.148 & 0.148 & -0.148 & 0.148 \\ 0.071 & 0.208 & 0.137 & -0.137 & -0.208 & -0.071 \\ -0.199 & 0.038 & 0.161 & 0.161 & 0.038 & -0.199 \end{pmatrix}$$

$$\text{WRITEPRN("f5invmatrix.dat")} := inf^{-1}$$

$$inf^{-1} \cdot struts = \begin{pmatrix} 1000 \\ -1000 \\ 2000 \\ 300 \\ -500 \\ 200 \end{pmatrix}$$

putting the above strut output through the inverse recovers the original cartesian vector.

Angles of the hardpoints from each axis:

It is necessary to look at the angles of the hardpoints with each axis. Because each hardpoint contains two flexures, the entire length between the attachments is not colinear. The end of the hardpoint between the flexure and attachment has an additional cosine term.

$$v := hpfixed - newmobile$$

Hardpoint vectors from input fields at beginning of report.

$$z := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Change to 100 or 010 or 001 for x,y,or z angles.

$i := 0..5$

$$\text{theta}_i := \text{acos}\left(\frac{v^{\langle i \rangle} \cdot z}{\left|v^{\langle i \rangle}\right| \cdot |z|}\right)$$

Use dot product with changing z vector.

$\text{RAD1} \cdot \text{theta}_i =$

35.47
35.491
35.518
35.475
35.501
35.522

Commutativity of transformation:

Look at effect on mobile coordinates of changing the order of combined translations and tilts.

$$\text{mobileTest1} := \left(\overrightarrow{E\left(0, \frac{0.05}{\text{RAD1}}, \frac{0.05}{\text{RAD1}}\right) \cdot \text{hpmobile} - T(1,1,-1)}\right)$$ Do translation first, then rotate.

$$\text{mobileTest2} := \left(E\left(0, \frac{0.05}{\text{RAD1}}, \frac{0.05}{\text{RAD1}}\right) \cdot \text{hpmobile} - T(1,1,-1)\right)$$ Do translation after rotations.

$$\text{mobileTest1} - \text{mobileTest2} = \begin{pmatrix} -0.001 & -0.001 & -0.001 & -0.001 & -0.001 \\ 0.001 & 0.001 & 0.001 & 0.001 & 0.001 \\ -7.612 \times 10^{-7} & -7.612 \times 10^{-7} & -7.612 \times 10^{-7} & -7.612 \times 10^{-7} & -7.612 \times 10^{-7} \end{pmatrix}$$

For the small motions, the difference between the two operations is about 1 micron, and so is of no concern here. However, in general, one must be careful, because if the translation is done first, then the rotation point must also shift to still be over the mirror vertex.

$$\text{mobileTest2} := \left(E\left(0, \frac{0.0}{\text{RAD1}}, \frac{-0.05}{\text{RAD1}}\right) \cdot \text{hpmobile} - T(0,.5,0)\right) - \text{hpfixed}$$

$i := 0..5$

$dxlength_i := \left| mobileTest2^{\langle i \rangle} \right|$

$$dxlength = \begin{pmatrix} 831.447 \\ 831.065 \\ 832.098 \\ 832.098 \\ 831.065 \\ 831.447 \end{pmatrix}$$

Appendix B:

Force-torque transformations for the MMT f/9 hexapod positioner.

This worksheet calculates matrices that convert between solid-body forces and torques applied to the output platform of a parallel positioner and forces acting on the individual strut actuators. It also converts forces applied to the center of gravity of the stuff attached to the output platform into strut loadings. This allows for example, the determination of hardpoint or hexapod stuts forces as a function of elevation.
This program is designed for platforms with 6 struts. The ends of each strut are connected to two platforms. The input plane is defined as the platform that is fixed to the structure and does not move as the actuators change length. The output (or mobile) plane articulates the work load.
One good application of this program is to use the strut platform as a force-torque sensor.

f9hexapod.mcd
scw: 2-11-97; update: 3-12-97

Work-points:

Work points define the upper and lower connection points of the struts to the input and output planes (or near the planes as desired). The coordinates of these points must have a common origin so that the strut vectors are calculated properly. Furthermore, one should know the position of this origin relative to the cg of the system or other location where one wants to apply forces to the output platform. (Although the latter coordinate offset is not needed until the end of the worksheet). These points must lie along the axis of each strut and can e.g. be the intersection points of adjacent struts.

$$cgorigin := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

This is used as a test for the cg force-moment transformation used at the end of the document. That transformation applied about the origin or this coordinate shift of the origin should be equivalent.

$$hpupper := READPRN(\text{"f9hexupper.prn"})^T$$

Read the workpoint coordinates near the moveable (or output) platform and transpose into column vectors. These are vectors from the origin to each connection point.

$$hplower := READPRN(\text{"f9hexlower.prn"})^T$$

Read the workpoint coordinates near the FIXED (or input) platform. These should have the same origin as the coordinates of the output platform.

$i := 0..5$

$hpupper^{\langle i \rangle} := hpupper^{\langle i \rangle} - cgorigin$

$hplower^{\langle i \rangle} := hplower^{\langle i \rangle} - cgorigin$

$wpvector := (hplower - hpupper)$

Workpoint vector elements (mm). The vectors are defined with their bases at the output platform.

$\left| hpupper^{\langle 4 \rangle} \right| = 223.75$

Distance of the output plane connection point from the origin.

$$hpupper = \begin{pmatrix} -193.77 & 193.77 & 193.77 & -193.77 & 0 & 0 \\ 111.875 & 111.875 & 111.875 & 111.875 & -223.75 & -223.75 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$i := 0..5$

$wplength_i := \left| wpvector^{\langle i \rangle} \right|$

$$wplength = \begin{pmatrix} 517.024 \\ 517.024 \\ 517.025 \\ 517.025 \\ 517.026 \\ 517.026 \end{pmatrix}$$

Strut lengths calculated from the input coordinates above.

$$wpvector = \begin{pmatrix} 193.77 & -193.77 & -11.18 & 11.18 & -182.59 & 182.59 \\ 98.962 & 98.962 & -217.293 & -217.293 & 118.332 & 118.332 \\ -469.014 & -469.014 & -469.014 & -469.014 & -469.014 & -469.014 \end{pmatrix}$$

Workpoint vectors (bases at output [upper] plane).

Projections of the strut vectors onto each axis:

$$xyz := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Axis unit vectors

$$wpdc := \left| \begin{array}{l} \text{for } j \in 0..2 \\ \quad \text{for } i \in 0..5 \\ \qquad dc_{j,i} \leftarrow \dfrac{wpvector^{\langle i \rangle} \cdot xyz^{\langle j \rangle}}{\left|wpvector^{\langle i \rangle}\right| \cdot \left|xyz^{\langle j \rangle}\right|} \\ dc \end{array} \right.$$

Calculate the direction cosines of the working point vectors using the dot product. In addition to projections onto the x,y,z axes, they serve as unit vectors for the hardpoints.

$$wpdc = \begin{pmatrix} 0.375 & -0.375 & -0.022 & 0.022 & -0.353 & 0.353 \\ 0.191 & 0.191 & -0.42 & -0.42 & 0.229 & 0.229 \\ -0.907 & -0.907 & -0.907 & -0.907 & -0.907 & -0.907 \end{pmatrix}$$

)irection cosine vectors for the struts. The base of the vector is at the output ılane working point.)

$$\overrightarrow{acos(wpdc) \cdot \frac{180}{\pi}} = \begin{pmatrix} 67.989 & 112.011 & 91.239 & 88.761 & 110.68 & 69.32 \\ 78.965 & 78.965 & 114.852 & 114.852 & 76.769 & 76.769 \\ 155.113 & 155.113 & 155.113 & 155.113 & 155.113 & 155.113 \end{pmatrix}$$

Vectors giving the angles that the struts make with the x,y,z axes (degrees).

Angles between adjacent struts:

$$acos\left(\frac{wpvector^{\langle 0 \rangle} \cdot wpvector^{\langle 1 \rangle}}{\left|wpvector^{\langle 0 \rangle}\right| \cdot \left|wpvector^{\langle 1 \rangle}\right|}\right) \cdot \frac{180}{\pi} = 44.02133$$

Angle between struts 1 and 2.

$$acos\left(\frac{wpvector^{\langle 1 \rangle} \cdot wpvector^{\langle 2 \rangle}}{\left|wpvector^{\langle 1 \rangle}\right| \cdot \left|wpvector^{\langle 2 \rangle}\right|}\right) \cdot \frac{180}{\pi} = 41.361$$

Angle between struts 2 and 3.

Torque sensitivites of each strut:

This section finds the contribution to the torque about the axes that each hardpoint detects.

$$wpN := \left| \begin{array}{l} \text{for } j \in 0..2 \\ \quad \text{for } i \in 0..5 \\ \qquad N_{j,i} \leftarrow xyz^{\langle j \rangle} \cdot \left(hpupper^{\langle i \rangle} \times wpdc^{\langle i \rangle}\right) \\ N \end{array} \right.$$

Find the contributions to the xyz axis torques that each strut senses. (Nmm) This is just the torque vector projected onto each axis.

$$wpN = \begin{pmatrix} -101.486 & -101.486 & -101.486 & -101.486 & 202.972 & 202.972 \\ -175.777 & 175.777 & 175.776 & -175.776 & 0 & 0 \\ -79.017 & 79.017 & -79.018 & 79.018 & -79.018 & 79.018 \end{pmatrix}$$

XYZ axis torque senstivities of each hardpoint for 1 N applied force.

This section creates a unitized torque sensitivity for an axisymmetric platform. This is useful because it produces a quantity proportional to the Euler rotation matrices.

$$unitupper := \left| \begin{array}{l} \text{for } j \in 0..2 \\ \quad \text{for } i \in 0..5 \\ \qquad u_{j,i} \leftarrow \dfrac{hpupper_{j,i}}{\left|hpupper^{\langle i \rangle}\right|} \\ u \end{array} \right.$$

Unitize the vector locations of the upper working points.

$$unitupper = \begin{pmatrix} -0.866 & 0.866 & 0.866 & -0.866 & 0 & 0 \\ 0.5 & 0.5 & 0.5 & 0.5 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\left|hpupper^{\langle 1 \rangle}\right| = 223.747$$

$$wpUN := \left| \begin{array}{l} \text{for } j \in 0..2 \\ \quad \text{for } i \in 0..5 \\ \qquad N_{j,i} \leftarrow xyz^{\langle j \rangle} \cdot \left(unitupper^{\langle i \rangle} \times wpdc^{\langle i \rangle}\right) \\ N \end{array} \right.$$

Unitized torque sensitivity for workpoints all the same distance away from the origin (i.e. axisymmetric strut geometry)

$$wpUN = \begin{pmatrix} -0.454 & -0.454 & -0.454 & -0.454 & 0.907 & 0.907 \\ -0.786 & 0.786 & 0.786 & -0.786 & 0 & 0 \\ -0.353 & 0.353 & -0.353 & 0.353 & -0.353 & 0.353 \end{pmatrix}$$

$M_{hp} := stack(wpdc, wpN)$

Stack the direction cosines and torque sensitivities into a single matrix.

$UM_{hp} := stack(wpdc, wpUN)$

Stack the unitized counterparts into a matrix.

Matrix that converts a strut force vector into the solid-body forces and moments about the origin. Muliply this matrix by vector (F1,F2,F3,...F6) and you obtain (Fx,Fy,Fz,Tx,Ty,Tz). The frist 3 rows are just the direction cosines, and the last three rows are in units of Nmm/(N of hardpoint force).

$$M_{hp} = \begin{pmatrix} 0.375 & -0.375 & -0.022 & 0.022 & -0.353 & 0.353 \\ 0.191 & 0.191 & -0.42 & -0.42 & 0.229 & 0.229 \\ -0.907 & -0.907 & -0.907 & -0.907 & -0.907 & -0.907 \\ -101.486 & -101.486 & -101.486 & -101.486 & 202.972 & 202.972 \\ -175.777 & 175.777 & 175.776 & -175.776 & 0 & 0 \\ -79.017 & 79.017 & -79.018 & 79.018 & -79.018 & 79.018 \end{pmatrix}$$

$$UM_{hp} = \begin{pmatrix} 0.375 & -0.375 & -0.022 & 0.022 & -0.353 & 0.353 \\ 0.191 & 0.191 & -0.42 & -0.42 & 0.229 & 0.229 \\ -0.907 & -0.907 & -0.907 & -0.907 & -0.907 & -0.907 \\ -0.454 & -0.454 & -0.454 & -0.454 & 0.907 & 0.907 \\ -0.786 & 0.786 & 0.786 & -0.786 & 0 & 0 \\ -0.353 & 0.353 & -0.353 & 0.353 & -0.353 & 0.353 \end{pmatrix}$$

Unitized matrix counterpart.

This matrix converts solid-body forces and torques into strut forces. Multiply this matrix by (Fx,Fy,Fz,Tx,Ty,Tz) and you obtain (F1,F2,F3,...F6).

$$M_{hp}^{-1} = \begin{pmatrix} 0.472 & 0.817 & -0.184 & -1.743 \times 10^{-3} & -8.901 \times 10^{-4} & -2.109 \times 10^{-3} \\ -0.472 & 0.817 & -0.184 & -1.743 \times 10^{-3} & 8.901 \times 10^{-4} & 2.109 \times 10^{-3} \\ 0.472 & -0.817 & -0.184 & 1.006 \times 10^{-4} & 1.954 \times 10^{-3} & -2.109 \times 10^{-3} \\ -0.472 & -0.817 & -0.184 & 1.006 \times 10^{-4} & -1.954 \times 10^{-3} & 2.109 \times 10^{-3} \\ -0.944 & 0 & -0.184 & 1.642 \times 10^{-3} & -1.064 \times 10^{-3} & -2.109 \times 10^{-3} \\ 0.944 & 0 & -0.184 & 1.642 \times 10^{-3} & 1.064 \times 10^{-3} & 2.109 \times 10^{-3} \end{pmatrix}$$

Applying forces about the cg:

This section transforms forces applied about the cg into equivalent forces and torques about the origin where the workpoints were defined. This allows one to calculate what forces the cg load applies to the struts for different orientations.

Input the coordinates of the cg from the origin where the workpoints were defined (taking care to preserve the sign of the original coordinate system).

$$r_{cg} := \begin{pmatrix} 0 \\ 0 \\ 8.1 \end{pmatrix}$$

Coordinate of the cg from the workpoint origin (mm). If this transform is used, make sure the "cgorigin" at the beginning of the worksheet is set to 0,0,0.

$$F_l := 160 \cdot 9.8$$

$$F_l = 1568$$

Load weight in N (careful of signs).

$$F_{cg}(F, el) := \begin{pmatrix} 0 \\ -F \cdot cos(el) \\ F \cdot sin(el) \end{pmatrix}$$

Load forces applied to the cg (Fx,Fy,Fz). Y is along elevation and points up for horizon pointing. This load points along +z and -y.

$$F_s := \left| \begin{array}{l} for \quad \psi \in 0..90 \\ \quad \left| \begin{array}{l} FT \leftarrow F_{cg}\left[F_l, \frac{\psi}{\left(\frac{180}{\pi}\right)}\right] \\ F \leftarrow {M_{hp}}^{-1} \cdot \begin{bmatrix} FT_0 \\ FT_1 \\ FT_2 \\ xyz^{\langle 0 \rangle} \cdot (r_{cg} \times FT) \\ xyz^{\langle 1 \rangle} \cdot (r_{cg} \times FT) \\ xyz^{\langle 2 \rangle} \cdot (r_{cg} \times FT) \end{bmatrix} \\ for \quad j \in 0..5 \\ \quad Q_{\psi, j} \leftarrow F_j \end{array} \right. \\ Q \end{array} \right.$$

Convert the forces applied to the cg to equivalent forces and torques about the workpoint origin then into strut forces. Note: if the cg was defined as the workpoint origin, then enter 0,0,0 for the coordinates of the cg above. NOTE: this transformation currently works only for FORCES applied to the cg, and NOT torques applied there. (If you use the coordinate offset at the beginning of the worksheet, the calc is valid for both forces and torques applied to the cg.)

Loop over elevation angle

$$\left(F_s{}^T\right)^{\langle 0\rangle} = \begin{pmatrix} -1303.846 \\ -1303.846 \\ 1282.99 \\ 1282.99 \\ 20.858 \\ 20.858 \end{pmatrix}$$

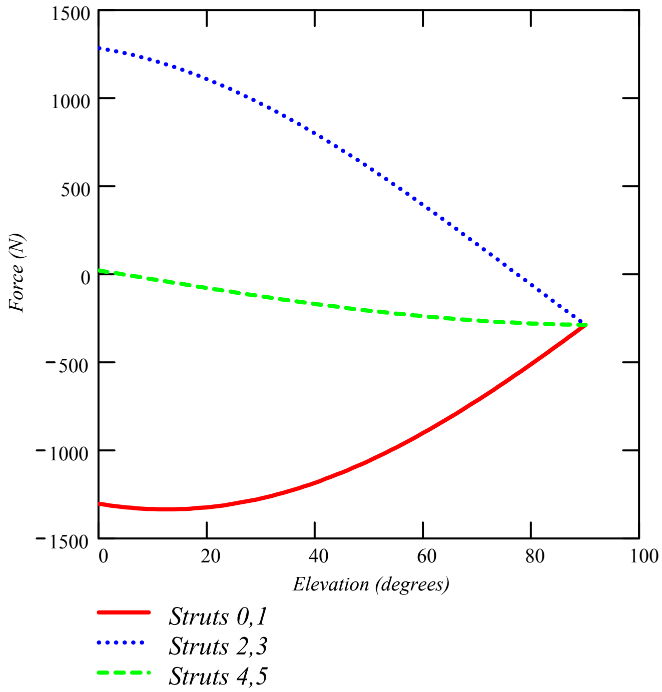
Strut forces at horizon pointing (N).
(-) = tension
(+) = compression

$$\left(F_s{}^T\right)^{\langle 90\rangle} = \begin{pmatrix} -288.084 \\ -288.084 \\ -288.086 \\ -288.086 \\ -288.085 \\ -288.085 \end{pmatrix}$$

Strut forces at zenith pointing (N).

$$hpupper = \begin{pmatrix} -193.77 & 193.77 & 193.77 & -193.77 & 0 & 0 \\ 111.875 & 111.875 & 111.875 & 111.875 & -223.75 & -223.75 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$i := 0 .. 90$



$I_i := i$

$F01_i := F_{s_{i,0}}$

$F23_i := F_{s_{i,2}}$

$F45_i := F_{s_{i,4}}$