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MMT Observatory

Smithsonian Institution &
The University of Arizona®

Secondary Mirrors Support
M2/F5 Hexapod Design
Kinematics Algorithm

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PROGRAMME : **MMT CONVERSION**
SECONDARY MIRRORS SUPPORT
M2/F5 HEXAPOD DESIGN

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1. SCOPE OF WORK

This note is devoted to derive the algorithms describing hexapod kinematics. A parametric approach is adopted in order to express mechanism layout as function of few basic elements of its geometry. The system is represented by a vector model in a Cartesian space. Both direct and inverse kinematic analyses are carried out.

2. APPLICABLE AND REFERENCE DOCUMENTS

2.1 Applicable Documents

- [1] G.Isella, "Preliminary Analysis of a Hexapod Pointing System for Space Applications", ESA STM-253, May 1994;

2.2 Reference Documents

- [2] W.H.Press et al., “Numerical Recipes”, Cambridge University Press, 1986;
- [3] J.R.Wertz, “Spacecraft Attitude Determination and Control”, 1978;

3. SYSTEM MODEL AND REFERENCE FRAME

3.1 System degrees of freedom

The hexapod is a mechanism made of three subsystems: the fixed platform, the actuators and the mobile platform. For the purposes of this analysis all these elements are modelled as rigid ones and the joints connecting them are considered as ideal, i.e. introducing pure kinematic relations.

There are 12 joints, each one restraining 3 degrees of freedom (dofs). Thus the overall number of system dofs is reduced to 6.

mobile platform	+	6
actuators	+	36 (6 x 6)
total	=	42
joints	-	36 (3 x 12)
grand total	=	6

Table 1: mechanism dofs

3.2 Fixed and Mobile Reference Frames

The fixed platform includes the fixed reference frame (X_F, Y_F, Z_F) to which all vectors and rotations are referred to. The reference frame origin is placed in one of the fixed joints. Z_F axis is perpendicular to the plane defined by the fixed platform. In this model, the fixed and mobile platforms are defined as the planes containing the actuator joints.

Being the hexapod a pointing system, it is convenient to introduce an auxiliary reference frame, to describe mechanism position in terms of mobile platform attitude and center of rotation position.

Mobile reference frame (X_M, Y_M, Z_M) origin lays on the axis of the mobile platform (perpendicular to the plane defined by the platform through its center) and defines mechanism center of rotation. Z_M axis is aligned with mobile platform one. X_M and Y_M axis are parallel, respectively, to X_F and Y_F ones (before performing any mobile platform rotation).

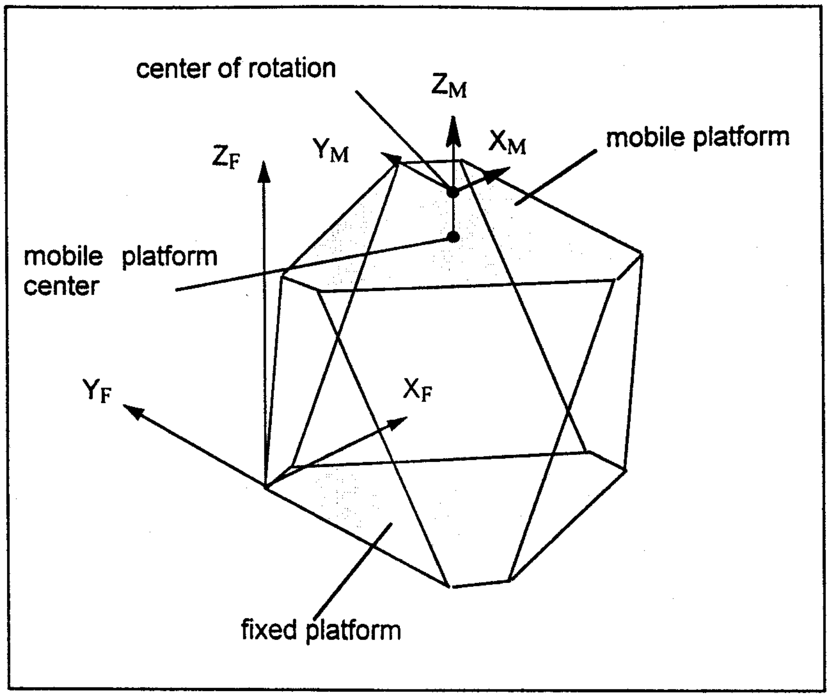

Figure 1: reference frames

3.3 Vector Model

System layout suggests to model each actuator with a three dimensional vector. The same approach is extended modelling both platforms by means of six vectors each. The overall mechanism is thus uniquely described with 18 vectors, named as follows:
fixed platform {v1, v2, v3, v4, v5, v6}; actuators {a, b, c, d, e, f}; mobile platform {α, β, γ, δ, ε, ζ}.

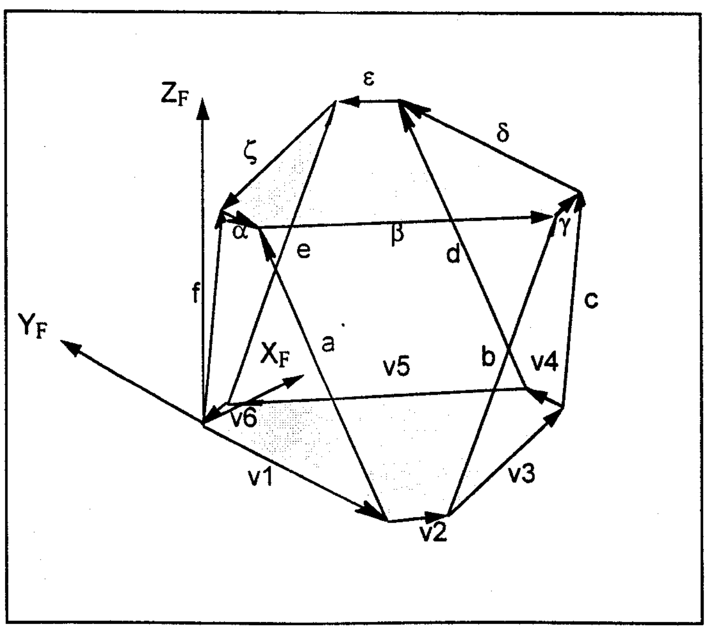

Figure 2: mathematical model of hexapod

The basic geometrical parameters which define hexapod geometry are hereafter listed.

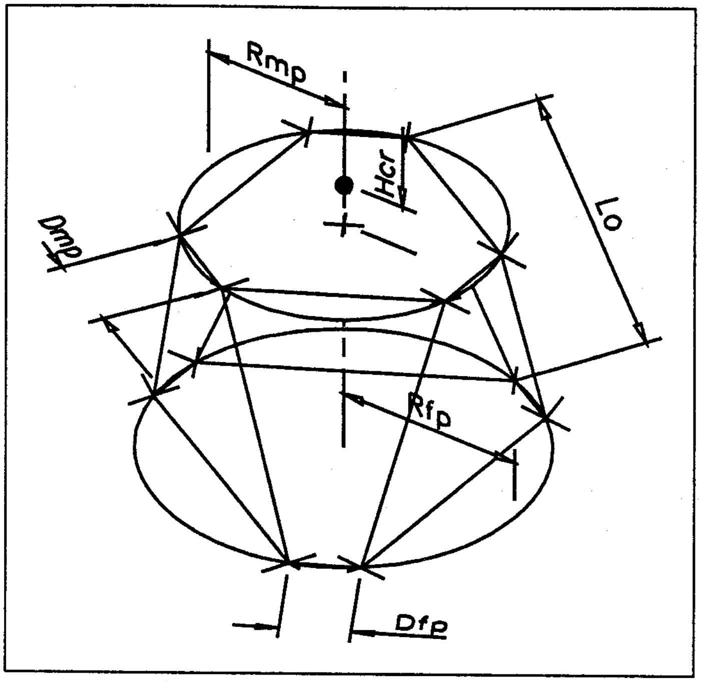

Figure 3: hexapod geometrical parameters

mobile platform radius	Rmp
mobile joints min. spacing	Dmp
fixed platform radius	Rfp
fixed joints min. spacing	Dfp
actuator length (azimuth=0 and elevation=0)	Lo
center of rotations height	Hcr

Table 2: geometrical parameters definitions

4. DIRECT KINEMATICS

4.1 Controlled Variables

Direct kinematic analysis model the actual functioning of the mechanism, where actuators lengths are the controlled variables. Position and attitude of the mobile platform have to be derived by knowing actuators lengths.

4.2 Kinematic Equations

Different equations sets can be written to define mechanism layout and internal restraints. Three independent closing equations (1) for hexapod lateral polygons are:

$$\begin{cases} a - c + \beta + \gamma = \mathrm{v}2 + \mathrm{v}3 \\ c - e + \delta + \varepsilon = \mathrm{v}4 + \mathrm{v}5 \\ e - a + \alpha + \zeta = \mathrm{v}6 + \mathrm{v}1 \end{cases} \qquad (1)$$

Other equations can be written to state that mobile platform vectors define a rigid body, i.e. both their lengths (2) and the angles among them are constants (3), and they all lay in a plane (4):

$$\begin{cases} |\alpha| = k_1 \\ |\beta| = k_2 \\ |\gamma| = k_1 \\ |\delta| = k_2 \\ |\varepsilon| = k_1 \end{cases} \qquad (2)$$

$$\begin{cases} \alpha \cdot \beta = k_3 \\ \beta \cdot \gamma = k_3 \\ \gamma \cdot \delta = k_3 \\ \delta \cdot \varepsilon = k_3 \end{cases} \qquad (3)$$

$$\begin{cases} \alpha \times \beta \cdot \gamma = 0 \\ \alpha \times \beta \cdot \varepsilon = 0 \\ \alpha \times \beta \cdot \zeta = 0 \end{cases} \qquad (4)$$

Finally, six more equations (5) are needed to assign actuators lengths, which are indeed the controlled variables:

$$\begin{cases} |a| = l_a \\ |b| = l_b \\ |c| = l_c \\ |d| = l_d \\ |e| = l_e \\ |f| = l_f \end{cases} \qquad (5)$$

4.3 Nonlinear System Solution

Equations (1) are vector equations in the unknown vectors $\{a\}=[x(a)\ y(a)\ z(a)]^T$, $\{c\}=[x(c)\ y(c)\ z(c)]^T$, $\{e\}=[x(e)\ y(e)\ z(e)]^T$ and scalar α, β, γ, δ, ε, ζ, while equations (2), (3), (4) and (5) are scalar ones.

There are 9 unknown vectors: a, c, e, α, β, γ, δ, ε, ζ for a grand total of 27 unknowns.

Equations (1), (2), (3), (4), and (5) can be simultaneously solved for the unknowns $\{x\} = [a, c, e, \alpha, \beta, \gamma, \delta, \varepsilon, \zeta]$.

Only equations (1) are linear, thus for the solution of nonlinear system a recursive procedure must be adopted, namely Newton-Raphson method [ref.2].

Given the set of N=27 nonlinear functions f_i to be zeroed, involving the variables x_i =1...27, a solution, if exist, is recursively approximated as:

$$x_i^{new} = x_i^{old} + \delta x_i , \qquad (6)$$

where δx_i is the solution of the linear system

$$\sum_{j=1}^{N} \alpha_{i,j} \delta x_j = \beta_i \qquad i=1...27 \qquad (7)$$

with $\alpha_{i,j} = \dfrac{\partial f_i}{\partial x_j}$ and $\beta_i = -f_i$.

The constant part of both the Jacobian $\alpha_{i,j}$ and coefficients β_i can be computed apart from the rest of the coefficients that depends on the values of the variables. As far as the recursive procedure converges, the solution $x = [a, c, e, \alpha, \beta, \gamma, \delta, \varepsilon, \zeta]$ is updated with the variations δx_i .

The remaining three unknown actuators [b, d, f] vectors can be obtained from the solution of the previous system by writing the closing equations for the lateral polygons not yet exploited:

$$\begin{cases} a - b + \beta = \mathrm{v2} \\ c - d + \delta = \mathrm{v4} \\ e - f + \zeta = \mathrm{v6} \end{cases} \qquad (8)$$

4.4 Attitude and Pointing Computation

When mechanism position is fully determined, mobile platform attitude is obtained as:

$$Z_M = \frac{\alpha \times \beta}{|\alpha \times \beta|}, \quad Y_M = \frac{\delta}{|\delta|} \quad \text{and} \quad X_M = Y_M \times Z_M \qquad (9)$$

Mobile platform center is given by:

$$vPC = v1 + a - \alpha/2 + \alpha c^* X_M \qquad (10)$$

where αc = Rmp * cos(arcsin(Dmp / 2 / Rmp)).

Center of rotation is computed as:

$$vCR = Hcr * Z_M \qquad (11)$$

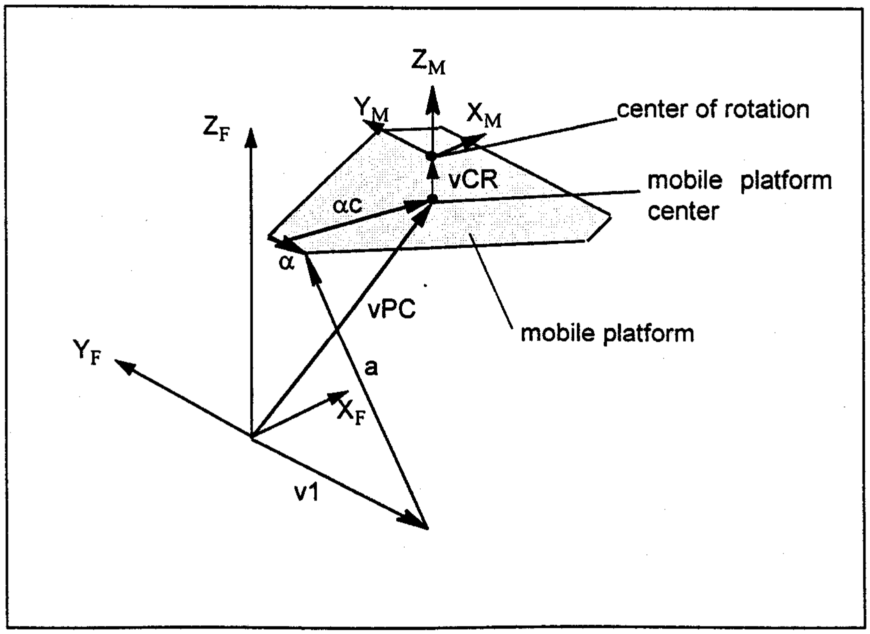

Figure 4: mobile platform center and center of rotations

The pointing angles of the mobile platform are computed as:

$$\text{elevation} \quad \lambda = \arccos(\, z(Z_M) \,)$$

$$\text{azimuth} \quad \rho = \arccos(\, x(Z_M) \,) \tag{12}$$

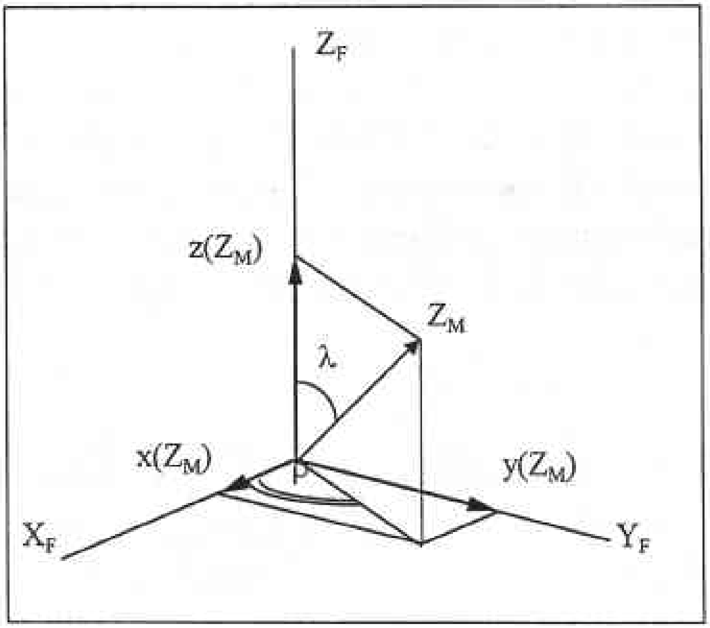

Figure 5: hexapod pointing angles

5. INVERSE KINEMATICS

5.1 Matrix of Rotations

Inverse kinematic analysis is intended to derive actuators lengths as function of mobile platform attitude. When the latter is expressed in terms of pointing coordinates, the displacements command sequence to be imposed to the actuators can be computed for any desired manoeuvre of the mechanism.

A feasible mode of functioning of the hexapod is obtained by imposing mobile platform rotations around a point, the center of rotations, which is kept fixed.

Euler angles represent a convenient way of describing such rotations [ref.3]. Among the available Euler angles rotation sequences, 3.2.3 one is chosen because allows rotation angles to be expressed directly as the pointing coordinates.

Euler rotation matrix [R323] (13)

$$\begin{bmatrix} \cos(\psi)\cdot\cos(\theta)\cdot\cos(\phi) - \sin(\psi)\cdot\sin(\phi) & \cos(\psi)\cdot\cos(\theta)\cdot\sin(\phi) + \sin(\psi)\cdot\cos(\phi) & -\cos(\psi)\cdot\sin(\theta) \\ -\sin(\psi)\cdot\cos(\theta)\cdot\cos(\phi) - \cos(\psi)\cdot\sin(\phi) & -\sin(\psi)\cdot\cos(\theta)\cdot\sin(\phi) + \cos(\psi)\cdot\cos(\phi) & \sin(\psi)\cdot\sin(\theta) \\ \sin(\theta)\cdot\cos(\phi) & \sin(\theta)\cdot\sin(\phi) & \cos(\theta) \end{bmatrix}$$

Being ρ and λ respectively azimuth and elevation angles, 3.2.3 rotation sequence is hereafter shown, where, as usual, the rotations order is assumed to be ϕ, θ and ψ:

First rotation directly performs azimuth manoeuvre:

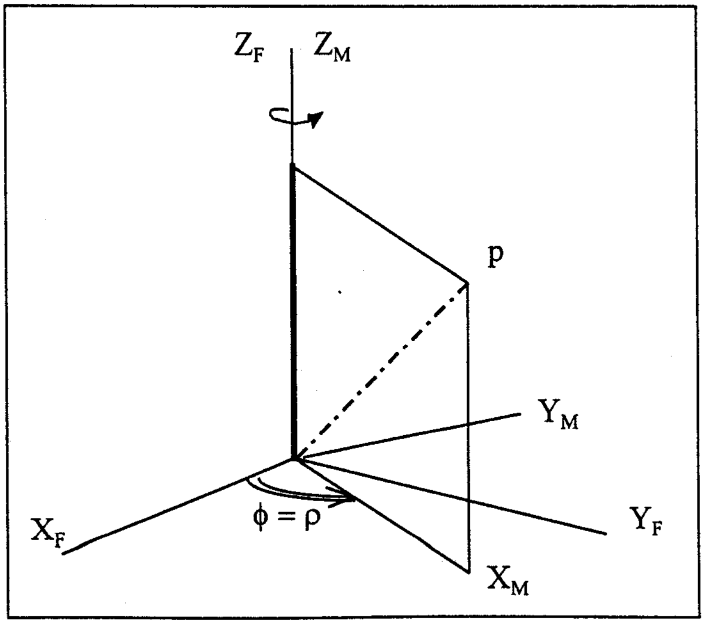

Figure 6: first Euler rotation (around axis 3, i.e. axis Z)

Second rotation around axis “2” executes elevation pointing:

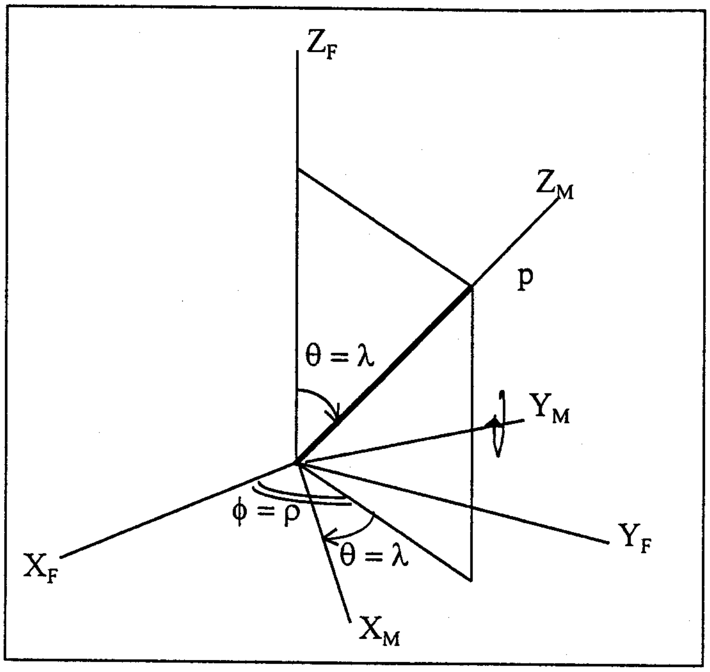

Figure 7: second Euler rotation (around axis 2, i.e. axis Y_M)

These two manoeuvres suffice in achieving platform pointing (Z_M axis).
With third rotation one can control platform own orientation around hexapod pointing direction. This last rotation is not uniquely determined when only azimuth and elevation coordinates are specified.
With this rotation sequence (3.2.3), to hold initial platform own orientation a rotation opposite to the azimuth angle must be executed. In fact, it can be demonstrated that $\psi = -\rho$ maximize mobile axis projection on fixed ones and minimize platform in plane rotation (see Fig.8).
Further ψ commands, if specified, can be added to this default one.

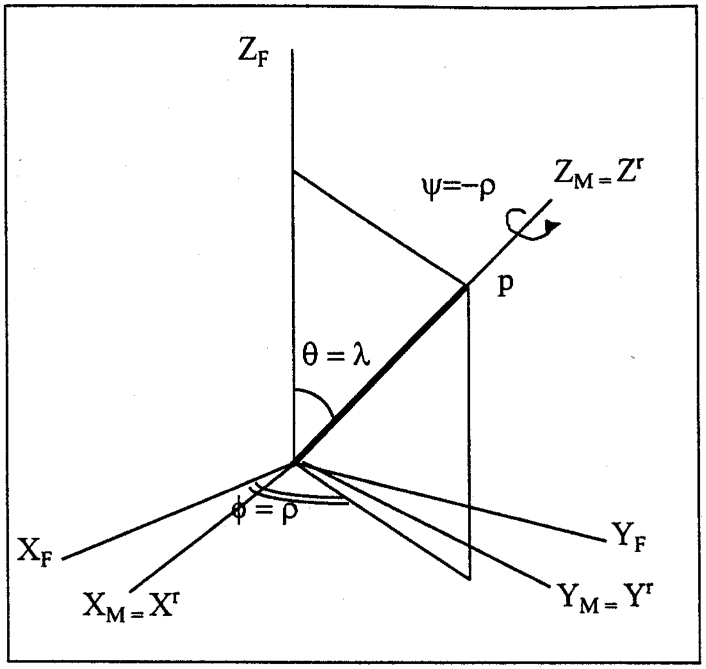

Figure 8: third Euler rotation (around axis 3, i.e. Z_M)

5.2 Mobile Platform Vectors Rotations

Matrix R323 elements are computed by substituting commanded values of ρ, λ and $-\rho$ to Euler angles ϕ, θ and ψ respectively.

Rotated vectors of the mobile platform, hereafter indicated by superscript "r" are computed by multiplying original ones by the rotation matrix:

$$\{\alpha\}^r = [R323]\ \{\alpha\} = [R323]\ [x(\alpha)\ y(\alpha)\ z(\alpha)]^T.$$
$$[...] \qquad (14)$$
$$\{\zeta\}^r = [R323]\ \{\zeta\} = [R323]\ [x(\zeta)\ y(\zeta)\ z(\zeta)]^T.$$

5.3 Mobile Platform Center Translation

If Hcr is not null, the rotation causes a translation of the mobile platform center (see Fig.9). The rotated position is computed as:

$$vPC^r = vPC + vCR - vCR^r = vPC + vCR - [R323]\ vCR \qquad (15)$$

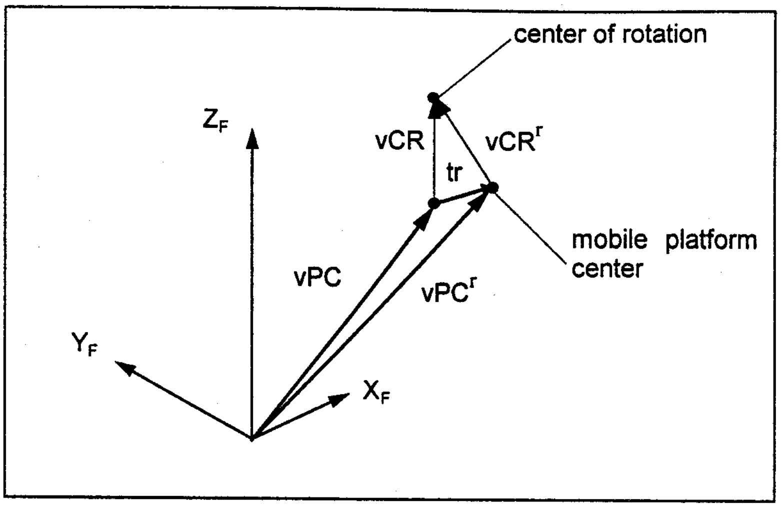

Figure 9: mobile platform center rotation

5.4 Mobile Reference Frame Computation

Mobile platform attitude can be calculated in terms of its reference frame components. This can be done by using equations (9) as done for inverse kinematics solution, by substituting rotated vectors α^r, β^r and δ^r instead of original ones.

$$Z_M{}^r = \frac{\alpha^r \times \beta^r}{|\alpha^r \times \beta^r|}, \quad Y_M{}^r = \frac{\delta^r}{|\delta^r|} \quad \text{and} \quad X_M{}^r = Y_M{}^r \times Z_M{}^r \qquad (16)$$

5.5 Actuators Length Computation

Rotated vector $\{a\}^r$ is computed as:

$$a^r = vPC^r - v1 + \alpha^r/2 - \alpha c\, X_M{}^r \qquad (17)$$

Vectors $\{c\}^r$ and $\{e\}^r$ are respectively given by:

$$c^r = a^r + \beta^r - v2 - v3 + \gamma^r \qquad (18)$$
$$e^r = v6 + v1 + a^r - \alpha^r - \zeta^r \qquad (19)$$

The remaining actuator vectors [b^r, d^r, f^r] are obtained from equations (8), again by using rotated vectors instead of original one. Then, actuators lengths are computed with equations (5) as the norm of actuator vectors.

6. FINAL REMARKS

Inverse kinematics equations for the hexapod mechanism allow to compute in closed form the actuator command sequence needed to perform any desired manoeuvre expressed in terms of payload pointing coordinates. By the other hand, for direct kinematics a numerical method is derived to compute mobile platform pointing angles from actuators elongations.

7. ANNEX A: ALTERNATIVE ROTATION SEQUENCE

The Euler attitude matrix [R3.2.3] allows to express pointing manoeuvre directly in terms of azimuth and elevation coordinates. The drawback is the need to compensate platform undesired in-plane rotation. However, it should be pointed out that third rotation component $\psi=-\rho$ does not actually represent an additional manoeuvre. In fact, all three angles φ, θ and ψ are indeed executed by a unique rotation, which results to be their composition.

This drawback can be avoided by choosing the rotation sequence [R2.1.3], in which ψ rotation is completely decoupled from the pitch and roll ones.

Euler rotation matrix [R213] (A.1)

$$\begin{bmatrix} \cos(\psi)\cdot\cos(\phi) + \sin(\psi)\cdot\sin(\theta)\cdot\sin(\phi) & \sin(\psi)\cdot\cos(\theta) & -\cos(\psi)\cdot\sin(\phi) + \sin(\psi)\cdot\sin(\theta)\cdot\cos(\phi) \\ -\sin(\psi)\cdot\cos(\phi) + \cos(\psi)\cdot\sin(\theta)\cdot\sin(\phi) & \cos(\psi)\cdot\cos(\theta) & \sin(\psi)\cdot\sin(\phi) + \cos(\psi)\cdot\sin(\theta)\cdot\cos(\phi) \\ \cos(\theta)\cdot\sin(\phi) & -\sin(\theta) & \cos(\theta)\cdot\cos(\phi) \end{bmatrix}$$

By the other hand, in this frame the pointing coordinates are expressed as a combination of pitch(ϕ) and roll(θ) angles:

$$\phi(\lambda,\rho) := \mathrm{atan}(\tan(\lambda)\cdot\cos(\rho)) \qquad \text{(A.2)}$$

$$\theta(\lambda,\rho) := -\mathrm{asin}(\sin(\lambda)\cdot\sin(\rho)) \qquad \text{(A.3)}$$

For this last reason sequence [R3.2.3] could be preferred to the latter one.