## MMTO Technical Memorandum \#03-3



Smithsonian Institution \&<br>The University of Arizona ${ }^{*}$

Status of Pointing of the 6.5 m MMT

T. Trebisky, C. Foltz, S. West

January 2003

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Tom Trebisky, Craig Foltz, Steve West<br>MMTO Observatory

January 29, 2003


#### Abstract

We present the results of a number of pointing runs spanning the last two and a half years. The pointing model is described and the coefficients from the model fits are presented. Our current pointing has an accuracy of about 1.6 arcseconds (rms), an error that includes a substantial ( $\sim 1 \operatorname{arcsec}$ ) component most likely due to low frequency oscillations of the absolute encoders. Experiments done by removing the collimation error determined by on-axis wavefront sensing indicate that the elevationdependent miscollimation is predominantly due to secondary tilt. This suggests that the tube flexure term is dominated by gravity induced twisting of the fixed hub rather than deformation of the OSS beams. We recommend that the telescope should be (1) pointed using the empirically determined pointing model; (2) corrected for collimation errors with the telescope re-pointed to account for the pointing error introduced in this process.


## Introduction

Pointing data have been obtained using several cameras mounted in the $\mathrm{f} / 9$ Top Box. Initial runs were done with an Apogee integrating CCD camera. More recently, the data have been collected with the Videoscope intensified camera using the 200 mm reimaging lens. The instrument rotator is disabled during pointing runs.

A typical pointing run includes the following steps:

1. The telescope operator determines the center of rotation of the field and marks it on the television monitor.
2. FK5 stars are acquired and then paddled manually onto the center of rotation. Typically, 30150 stars are acquired, generally above 15 degrees elevation.
3. The calculated azimuth and elevation, current coordinates, time and pointing errors (the paddled offsets) are recorded in a file on the operator's Linux PC via a button push on the operator's TCS GUI. The format of the raw data file is given in Appendix A.
4. The pointing file is converted into TPOINT ${ }^{\mathrm{TM}}$ input format via a short perl script (see Appendix A). Analysis of the pointing errors and construction of a new model is done off-line using the TPOINT software package.
5. The new pointing coefficients are loaded into the TCS code.

## Pointing Model

The standard TPOINT 'azel' model ${ }^{1}$ is used. The baseline model includes six geometric terms; other azimuth- and elevation-dependent terms can be optionally included. Prior to spring 2002, the pointing errors were dominated by non-repeatability in the $\mathrm{f} / 9$ secondary support system so only the six geometric terms were used. Following the stiffening of the secondary cell, two elevation-dependent terms were added to the model. No significant azimuth dependence has been found in any of the pointing runs.

The eight terms and their brief descriptions are listed below. More complete descriptions are given in Appendix B.

Pointing model terms:
IA = Azimuth encoder offset
IE = Elevation encoder offset
$\mathrm{NPAE}=\mathrm{Az} / \mathrm{El}$ non-perpendicularity, causing azimuth shift
CA $=$ Left/Right collimation error, causing left-right shift
AN $=$ North/South azimuth axis misalignment
AW = East/West azimuth axis misalignment
TF = Sinusoidal tube flexure -- coefficient of cosine(elevation)
TX = Empirical tube flexure -- coefficient of cotangent(elevation)
All units are in arcseconds.

## Results

Details on nine pointing runs are presented in the table below.

| Date | IA | IE | NPAE | CA | AN | AW | TF | TX | rms N $_{\text {stars }}$ |  |
| :--- | :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $1 / 07 / 2003$ | 1202.56 | 55.13 | 1.61 | -45.76 | -3.22 | -9.71 | +2.04 | -6.24 | 1.67 | 143 |
| $9 / 16 / 2002$ | 1202.29 | 66.34 | 2.02 | -3.93 | 0.27 | -12.88 | -0.29 | -4.95 | 0.99 | 60 |
| $3 / 21 / 2002$ | 1204.53 | 76.04 | -1.66 | 29.63 | -2.61 | -11.38 | -9.08 | -4.43 | 0.97 | 33 |
| $4 / 09 / 2001$ | 1164.61 | 33.56 | -33.18 | 116.58 | 5.62 | -35.47 | 0.00 | 0.00 | 7.30 | 76 |
| $4 / 07 / 2001$ | 1186.85 | 54.03 | -13.38 | 146.74 | 1.80 | -26.09 | 0.00 | 0.00 | 8.40 | 40 |
| $9 / 01 / 2000$ | 1188.83 | -5.53 | -8.35 | 3.13 | -3.49 | -26.83 | 0.00 | 0.00 | 8.50 | 123 |
| $7 / 19 / 2000$ | 1179.57 | 68.89 | -20.14 | 46.35 | 0.97 | -18.40 | 0.00 | 0.00 | 8.20 | 87 |

As noted above, prior to the March 2002 run, the pointing errors were dominated by large and nonrepeatable flexure in the $f / 9$ secondary cell. These models did not include elevation-dependent terms and are included here only for completeness. However, note that the removal of the compliance in the $\mathrm{f} / 9$ secondary cell in 2001 reduced the rms pointing error by nearly an order of magnitude.

The rms error contains two known components from sources other than uncorrected flexure. The first is the error in the centering of the stellar image on the center of rotation. This is dependent on seeing and, to some extent, brightness of the star. We estimate it to be of order 0.25 arcseconds. A second component, which is certainly present in the 2003 data, is due to a slow $(\sim 0.01 \mathrm{~Hz}$ at typical
tracking speed) oscillation of the absolute encoders with about 1 arcsec amplitude ${ }^{2}$. This was only first characterized in the autumn of 2002 so it is not known whether it was present in earlier data. The increase in the rms error from September suggests that the onset of this yet-to-be-diagnosed oscillation may be at fault.

Why use two elevation-dependent terms? The answer is that a simple term proportional to the cosine of the elevation does not approximate the elevation dependence well. This can be seen by considering the results of fitting one dataset (March 21, 2002) with a six, seven and eight term model. The coefficients and associated errors for the six-term model are tabulated below.

|  | Coeff | change | value | sigma |
| ---: | :---: | ---: | ---: | ---: |
| 1 | IA | +0.124 | +1205.27 | 17.720 |
| 2 | IE | -1.564 | +87.77 | 0.917 |
| 3 | NPAE | +0.526 | -0.97 | 17.142 |
| 4 | CA | -0.095 | +28.66 | 23.374 |
| 5 | AN | -0.003 | -2.29 | 1.063 |
| 6 | AW | +0.036 | -11.28 | 1.066 |
| Sky RMS $=$ | 5.65 arcsec |  |  |  |

A scatter plot of pointing error, shown in the left panel of Figure 1, shows a strong elevationdependent effect. Plotting the elevation errors as a function of elevation (or more precisely, the errors in zenith distance as a function of zenith distance) shows this dependence clearly in the right panel:


Figure 1. Scatter plot of pointing errors and elevation dependence of errors for a six-term geometric pointing model.
Adding a term proportional to cosine(elevation) results in the following seven-term model:


Although the resulting fit is much improved, there is still evidence for elevation dependence, as seen in Figure 2.


Figure 2. Results for a seven-term model with a simple cosine(elevation) flexure term.

Finally, a term proportional to cotangent(elevation) is included with the following result:

|  | coeff | change | value | sigma |
| ---: | :---: | ---: | ---: | ---: |
| 1 | IA | +0.000 | +1204.53 | 3.027 |
| 2 | IE | -0.000 | +76.04 | 0.772 |
| 3 | NPAE | +0.000 | -1.66 | 2.928 |
| 4 | CA | -0.001 | +29.63 | 3.993 |
| 5 | AN | -0.000 | -2.61 | 0.183 |
| 6 | AW | -0.000 | -11.38 | 0.182 |
| 7 | TF | -0.000 | -9.08 | 1.795 |
| 8 | TX | +0.000 | -4.43 | 0.429 |
| Sky | RMS $=$ | 0.97 | arcsec |  |



Figure 3. Results for an eight-term model including cotangent(elevation) flexure term.

As seen in Figure 3, the scatter plot is symmetric in azimuth and elevation and any residual elevation dependence is small.

## The Nature of the Elevation Flexure

As described in MMT Technical Memo \#03-01 ${ }^{3}$, a tool for correcting on-axis elevation-dependent collimation errors has been devised and is in routine use. This tool is named "elcoll." The basis for the correction is analysis of wavefront sensor data to measure the repeatable variations of on-axis coma and corrects them by applying tilt to the $f / 9$ secondary mirror. The tool then repoints the telescope to correct for the mispointing introduced by the tilt.

A pointing run was carried on January 13, 2003. For this run, elcoll was used to correct the collimation prior to moving the mount to center the star. The results for this run are given in the table below.

| Date | IA | IE | NPAE | CA | AN | AW | TF | TX | rms | $\mathbf{N}_{\text {stars }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1 / 07 / 2003$ | 1202.56 | 55.13 | 1.61 | -45.76 | -3.22 | -9.71 | +2.04 | -6.24 | 1.67 | 143 |
| $1 / 13 / 2003$ | 1204.20 | 43.18 | 3.29 | -34.69 | -3.43 | -10.11 | -25.65 | -5.14 | 1.28 | 100 |

As can be seen, the most significant change in this run, as compared to the January 7, 2003 model, was a significant increase in TF, the cosine(elevation) term. Since correcting collimation and then repointing to compensate wavefront tilt increased the TF term rather than reducing it, the implication is that the basic OSS structural members are more tightly coupled to the elevation encoder than is the orientation of the fixed hub which is supported by the spiders. So, constructing a pointing model without elcoll produces a basic model that is more independent of which secondary is attached. This suggests that the pointing model and collimation model be separated. We recommend that the telescope be pointed in the following way: (1) Point the telescope using a pointing model derived without using elcoll. (2) Correct the telescope for collimation errors using elcoll or the wavefront sensor. We note that the substantially increased mass of the $\mathrm{f} / 5$ secondary package may require us to modify this procedure.

## References

${ }^{1}$ "TPOINT - A Telescope Pointing Analysis System," Ver. 5.4-3 for Unix, (Tpoint Software: Drayton, Abington, Oxfordshire) 1998.
${ }^{2}$ MMTO Bi-Monthly Summary, September-October 2002.
${ }^{3}$ West, S.C. "Fall 2002 f/9 Optical Performance of the 6.5 m MMT Analyzed with the Top Box ShackHartmann Wavefront Sensor," MMT Technical Memo \#03-1, 2003

## Appendix A - Notes on pointing data analysis

```
Collect pointing data using the SAVE button in the scope GUI.
Data used to go to /home/tom/catalogs/pointing.dat
Now it goes to /mmt/Logs/pointing/pointing.dat
Copy this to /mmt/pointing/archive/pointing.dat.020104
    (use current year/month/day)
make a new script "prep19" or whatever, that reads
this file and emits mmt19.dat.
Get humidity, pressure, temperature from the log,
and use tp_calc to convert units.
```

To run tpoint in the simplest way, do this:
tpoint
indat mmt7.dat
call altaz
fit
fit
call a6
The "call altaz" model asks for a 6 term model with the following
terms:
AN - Azimuth Axis misalignment, North-South
IA - Azimuth index error (encoder offset)
IE - Elevation index error (encoder offset
CA - Left-Right Collimation error
NPAE - Az-El Non-perpendicularity
AW - Azimuth Axis misalignment, East-West
other terms to try:
TF - classic tube flexure $d(E l)=-T F \cos (E l)$
$T X$ - empirical tube flexure $d(E l)=-T X \cot (E 1)$
The variables on the Tpoint graphs are:
A = Azimuth (labeled N E S W N)
S = left-right on sky
Z = Zenith distance
$\mathrm{V}=$ result changes Az/El non-perpendicularity
AM = Axis misalignment (from GAM command)
GSMAP is very useful, showing residuals in a polar projection.
(in the case of data from 1/4/2002, it showed big
residuals for stars to the East (pointing into wind??)
and near horizon).

```
----------------------------------------------------------------------
As of Jan, 2002, the format of our "raw" pointing file
consists of records in the following format.
Relative to the descriptions below, this adds the records
SVEL and SERR, which are servo velocity and error, if these
values are high they would indicate that the mount was not
tracking the star well, and the pointing record would be suspect.
CAT: FK5-0261 6:52:47.337 +33:57:40.44 -00.017 -004.81 3.60 A2
J2000.0
RAA: 87.1412019 36.8769462
AA: 87.1458603 37.8417652
TAA: 87.1537029 37.4237240
LST: 06:44:37.65
CRD: 06:52:38.25 +33:58:07.86
SVEL: 0.0020 -0.0545
SERR: 1 -66
On 6/21/2000, we changed to the following format for our pointing data files:
```

```
CAT: FK5-1468 17:55:25.191 +26:03:00.00 +00.033 +000.61 5.46 F5
```

CAT: FK5-1468 17:55:25.191 +26:03:00.00 +00.033 +000.61 5.46 F5
J2000.0
RAA: 84.3852296 166.3725361
AA: 84.2551857 167.1299756
TAA: 84.2436080 167.0247462
LRA: }\quad0174932.72 00 00 00.005 00 00 00.01
CRD: 17:55:16.91 +25:53:49.52
The basic rules are this:
Each line begins with a unique alphanumeric tag.
Once these stabilize, new tags can be added as desired, but existing
tags with remain unchanged. These are as follows:
CAT: The catalog record that was used to select the object
(at present these are from our FK5 catalog).
RAA: raw alt/az coordinates based on the catalog RA and DEC.
( NEW! added 6/20/2000 ) (includes refraction).
AA: alt/az coordinates with existing pointing corrections.
(Note:
AA: here is what was the first two columns of AA
in the old format, the last two columns are now TAA).
TAA: alt/az coordinates (T for telescope).
LRA: is poorly formatted (should have colons).
on top of that the RA and DEC at the end of the record
are bogus. It is almost entirely bogus for the new scheme
and will be replaced by an LST record.

```
```

        The first thing (the 0) is the record in the catalog file.
        The next 3 are the lst (without colons).
    LRA: 0 17 49 32.72 00 00 00.005 00 00 00.01
LRA: 0 19 33 10.14 00 00 00.005 00 00 00.01
On 6/22/2000, we changed to the following format for our pointing
data files:
CAT: FK5-1468 17:55:25.191 +26:03:00.00 +00.033 +000.61 5.46 F5
J2000.0
RAA: 84.3852296 166.3725361
AA: 84.2551857 167.1299756
TAA: 84.2436080 167.0247462
LST: 17:49:32.72
CRD: 17:55:16.91 +25:53:49.52
The main changes are that there is now a colon on the CAT record,
and a sensible LST record now replaces the old LRA record.
(notice that the old FDC code can still produce the ancient
record formats (maybe), but we never expect to use it).

```
```

Prior to 6/21/200, we used the following format

```
Prior to 6/21/200, we used the following format
for our pointing files:
for our pointing files:
AA: 49.0748846 98.6083457 49.1012429 98.6189547
AA: 49.0748846 98.6083457 49.1012429 98.6189547
LRA: 0 19 33 10.14 00 00 00.005 00 00 00.01
LRA: 0 19 33 10.14 00 00 00.005 00 00 00.01
CRA: 22:24:57.93
CRA: 22:24:57.93
CDEC: +18:08:37.44
CDEC: +18:08:37.44
---------------------------------------------
The following perl script was used to prepare this data
for input to TPOINT:
#!/usr/bin/perl
# Convert our mess of data into the format tpoint wants.
# Tpoint wants:
# AZ-observ, EL-observ, AZ-raw, EL-raw.
#$pfile = "./point4.dat";
$pfile = "./point_6_7_2000";
open PF,$pfile or die "Cannot open input file\n";
foreach ( <PF> ) {
    next if m/^$/;
    next if m/^#/;
    next unless m/^AA:/;
```

```
        chomp;
        @stuff = split;
        # discard $stuff[0]
        $azo = 180.0 - $stuff[4];
    $elo = $stuff[3];
    $azr = 180.0 - $stuff[2];
    $elr = $stuff[1];
    print "$azo $elo $azr $elr\n";
}
On UT 1/7/2003, we got a 143 star pointing run.
prep10 >mmt10.out
tpoint
indat mmt10.out
call altaz
use TF
use TX
fit
fit
call a6
gave us 1.67 arc-second pointing.
```

Also useful are:
gscat a to get an alt/az residual scatter plot.
slist to get a star by star list of residuals.
As of $1 / 16 / 2003$, an entry in the raw pointing data file
collected when the operator hits "SAVE" is:
CAT: FK5-0339 9:00:38.390 +41:46:57.98-03.932-024.59 3.97 F5
J2000.0
RAA: 79.9167443 358.5670507
AA: $\quad 79.9314405 \quad 358.8462968$
TAA: 79.9288006 -1.1314825
LST: 09:02:11.55
RRD: $\quad 9.0136458 \quad 41.7724120$
ARD: $\quad 9.014110641 .7701727$
CRD: 09:00:35.583 41:47:11.762
SVEL: 0.0000 -0.0177
SERR: -2 -11
The above raw entry becomes:
181.131482579 .9288006 -178.5670507 79.9167443
The actual file that tpoint inhales has the following header:
! Here is pointing data from 1/13/2003
! MMT pointing data in tpoint format 4

A-iv

```
! Next follows the header record. (Tpoint manual p. 23)
! (TPOINT run parameters record)
! First 3 numbers are latitude (deg,min,sec)
! The next 3 are UTC date (year, month, day)
! The next 2 are Temp and Pressure
! The last 2 are Elevation and Rel. Humidity
!
! 46.8 degrees F => 7 Celsius ?
! 555 mm of mercury => 727 mB ?
MMT pointing data 1-13-2003
+31 41 19.6 2003 01 13 +6.7 738.0 2608.0 0.30
! These data are observed AZ/EL followed by Raw AZ/EL
! transformed AZ into tpoint convention (South = 0, East=90)
181.1314825 79.9288006 -178.5670507 79.9167443
.... more and more stars follow.
Here is the perl script in use as of 1/16/2003
It loops thru the raw pointing file, extracting stuff into
a form palatable to tpoint as follows:
#!/usr/bin/perl
# prep11 - tpoint conversion
# (for files after 6/20/2000)
# run as ./prep11 >mmt11.dat
# Tpoint wants:
# AZ-observ, EL-observ, AZ-raw, EL-raw.
$pname = "pointing_1_13_2003";
$pdir = "/mmt/pointing/archive/";
$pfile = $pdir . $pname;
$hfile = "./TPOINT.HEADER";
open HF,$hfile or die "Cannot open header file\n";
foreach ( <HF> ) {
    print;
}
open PF,$pfile or die "Cannot open input file\n";
foreach ( <PF> ) {
    next if m/^$/;
    next if m/^#/;
    chomp;
    if ( /^RAA:/ ) {
                @stuff = split;
                $azr = 180.0 - $stuff[2];
                $elr = $stuff[1];
    } elsif ( /^TAA:/ ) {
```

```
        @stuff = split;
        $azo = 180.0 - $stuff[2];
        $elo = $stuff[1];
        print "$azo $elo $azr $elr\n";
    }
}
\# THE END
```


## Appendix B - Relevant TPOINT Coefficients

The following descriptions are taken from the TPOINT manual ${ }^{1}$
AN Azimuth Axis Misalignment North-South AN

## DESCRIPTION :

In an altazimuth mount, misalignment of the azimuth axis north-south: rotation about a horizontal east-west axis equal to coefficient AN.

## FORMULA :

$$
\begin{aligned}
& \Delta A=+\mathrm{AN} \sin A \tan E \\
& \Delta E=+\mathrm{AN} \cos A
\end{aligned}
$$

(Take care with sign conventions: inside TPOINT, azimuth increases anticlockwise.)

## NOTES :

AN is one of the six purely geometrical terms that affect all altazimuth mounts, the others being IA, IE, CA, NPAE and AW.

If AN is positive, the pole of the mounting is north of the zenith.
The other (i.e. east-west) component of the azimuth axis misalignment is the term AW.

AW
Azimuth Axis Misalignment East-West
AW

In an altazimuth mount, misalignment of the azimuth axis east-west: rotation about a horizontal north-south axis equal to coefficient AW.

## FORMULA :

$$
\begin{aligned}
& \Delta A=-\mathrm{AW} \cos A \tan E \\
& \Delta E=+\mathrm{AW} \sin A
\end{aligned}
$$

(Take care with sign conventions: inside TPOINT, azimuth increases anticlockwise.)

## NOTES :

AW is one of the six purely geometrical terms that affect all altazimuth mounts, the others being IA, IE, CA, NPAE and AN.

If AW is positive, the pole of the mounting is west of the zenith.
The other (i.e. north-south) component of the azimuth axis misalignment is the term AN.

## CA

Left-Right Collimation Error
CA

## DESCRIPTION :

In an altazimuth mount, the collimation error is the non-perpendicularity between the nominated pointing direction and the elevation axis. It produces a left-right shift on the sky that is constant for all elevations.

## FORMULA :

$$
\Delta A=+C A \sec E
$$

(Take care with sign conventions: inside TPOINT, azimuth increases anticlockwise.)

## NOTES :

CA is one of the six purely geometrical terms that affect all altazimuth mounts, the others being IA, IE, NPAE, AN and AW.

A non-zero CA on its own means there is an area around the zenith inside which the telescope cannot point (irrespective of velocity and acceleration limits close to the zenith).

Collimation error has many causes - misalignment of the optics, an inaccurately centered eyepiece graticule and so on.

## IA

Index Error in Azimuth
IA

## DESCRIPTION :

Azimuth index error in an altazimuth mount: the zero-point error in $A$.

## FORMULA :

$$
\Delta A=+\mathrm{IA}
$$

(Take care with sign conventions: inside TPOINT, azimuth increases anticlockwise.)

## NOTES :

On many altazimuth telescopes, IA is arbitrarily set at the start of each night by pointing at a calibration star, and on telescopes without absolute encoders or readouts there is no choice; under these circumstances the IA value returned by TPOINT has no lasting significance. However, note that it is not the same thing as a horizontal error on the sky, such as might be caused by a badly adjusted primary mirror; the latter, addressed by the CA term, would produce the same image displacement at all elevations, whereas IA has a lessening effect as the zenith is approached.

IA is one of the six purely geometrical terms that affect all equatorial mounts, the others being IE, CA, NPAE, AN and AW.

## IE <br> Index Error in Elevation <br> IE

## DESCRIPTION :

Elevation index error in an altazimuth mount: the zero-point error in $E$.

## FORMULA :

$$
\Delta E=+\mathrm{IE}
$$

## NOTES :

IE produces a fixed vertical offset of the image. On many altazimuth telescopes it is arbitrarily set at the start of each night by pointing at a calibration star, and on telescopes without absolute encoders or readouts there is no choice; under these circumstances the IE value returned by TPOINT has no lasting significance. It is indistinguishable from various other vertical shifts, such as those caused by a badly adjusted primary mirror.

IE is one of the six purely geometrical terms that affect all altazimuth mounts, the others being IA, CA, NPAE, AN and AW.

## NPAE

## DESCRIPTION :

In an altazimuth mount, if the azimuth axis and elevation axis are not exactly at right angles, horizontal shifts of the image occur that are proportional to $\sin E$.

## FORMULA :

$$
\Delta A=+ \text { NPAE } \tan E
$$

## NOTES :

NPAE is one of the six purely geometrical terms that affect all altazimuth mounts, the others being IA, IE, CA, AN and AW.

The image displacement produced by NPAE is zero on the horizon and reaches a maximum at the zenith.

Just as $E$ is always positive, the correction for NPAE never changes sign. Furthermore, because there is more sky at lower elevations than near the zenith there are likely to be fewer pointing observations in a given elevation-band near the zenith than in a similar band nearer the horizon. These two effects conspire so that the NPAE effect tends to be very poorly sampled when pointing tests are made, and hence the NPAE coefficient is apt to be poorly determined. In some cases, it can be better to omit NPAE from the model and rely on other terms mopping up any NPAE-like effect that are present.

A non-zero NPAE on its own means there is an area around the zenith inside which the telescope cannot point (irrespective of velocity and acceleration limits close to the zenith).

## TF

Tube Flexure (sine)
TF

## DESCRIPTION :

Classical tube flexure: change in zenith distance proportional to $\sin \zeta$.

## FORMULA :

$$
\Delta E=-\mathrm{TF} \cos E
$$

In equatorial spherical coordinates:

$$
\begin{aligned}
& \Delta b=+\mathrm{TF} \cos \varphi \sin h \sec \delta \\
& \Delta \delta=+\mathrm{TF}(\cos \varphi \cos h \sin \delta-\sin \varphi \cos \delta)
\end{aligned}
$$

## NOTES :

This is the classic tube flexure model, which assumes that the telescope obeys Hooke's Law. In practice there is often a rapid increase in the vertical displacement towards the horizon, and it is sometimes found that a tangent law is a better approximation that a sine law: see the TX term.

TX

## DESCRIPTION :

Empirical tube flexure: change in zenith distance proportional to $\tan \zeta$.

## FORMULA :

$$
\Delta E=-\mathrm{TX} \cot E
$$

In equatorial spherical coordinates:

$$
\begin{aligned}
& \Delta h=+ \text { TX } \cos \varphi \sin h /((\sin \delta \sin \varphi+\cos \delta \cos h \cos \varphi) \cos \delta) \\
& \Delta \delta=+ \text { TX } \cos \varphi \cos h \sin \delta-\sin \varphi \cos \delta /((\sin \delta \sin \varphi+\cos \delta \cos h \cos \varphi) \cos \delta)
\end{aligned}
$$

## NOTES :

The classic tube flexure model, TF, which assumes that the telescope obeys Hooke's Law, is a sine rather than a tangent law. In practice there is often a rapid increase in the vertical displacement towards the horizon, and it is sometimes found that the present term, TX, is a better approximation than TF.

Note that TX will compensate for any scaling error in the refraction corrections.

