

Reduced-order Models Fit to Open-loop Data for the MMT Azimuth Axis

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Introduction

The full-order mechanical model for the MMT azimuth drive system reported earlier is large and complex. For the purposes of design study, a simplified version of the drive response is appropriate to use in frequency-response based controller design. In this report the model-order reduction process is described, along with discussion of the model fitting to the measured response data.

Model Order Reduction

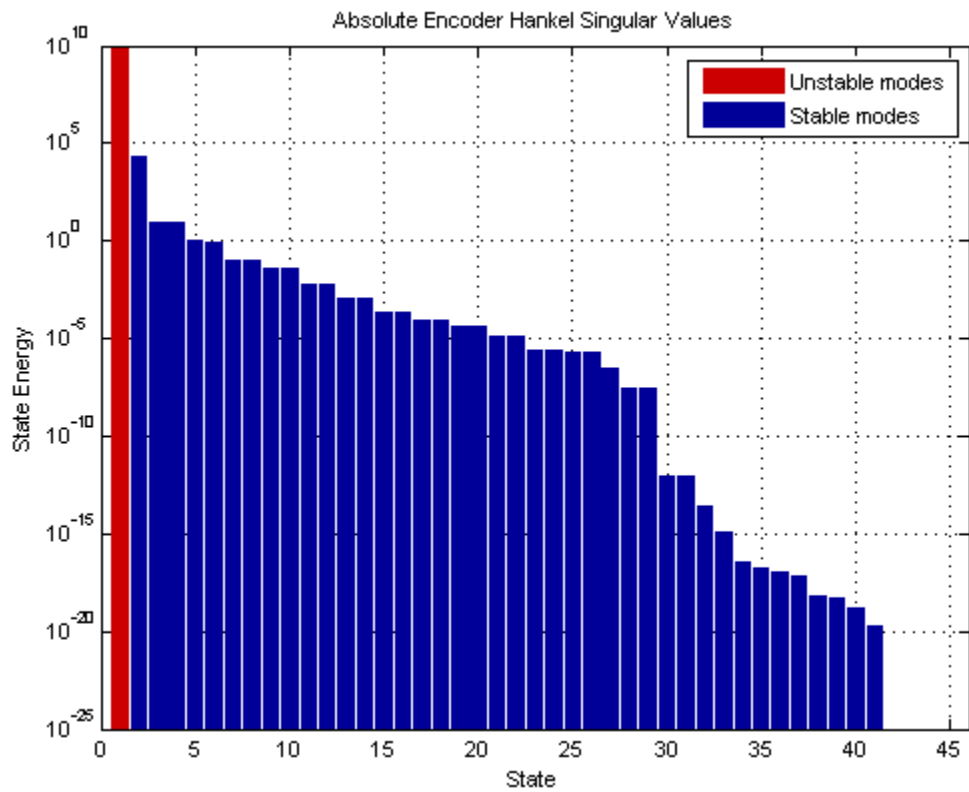
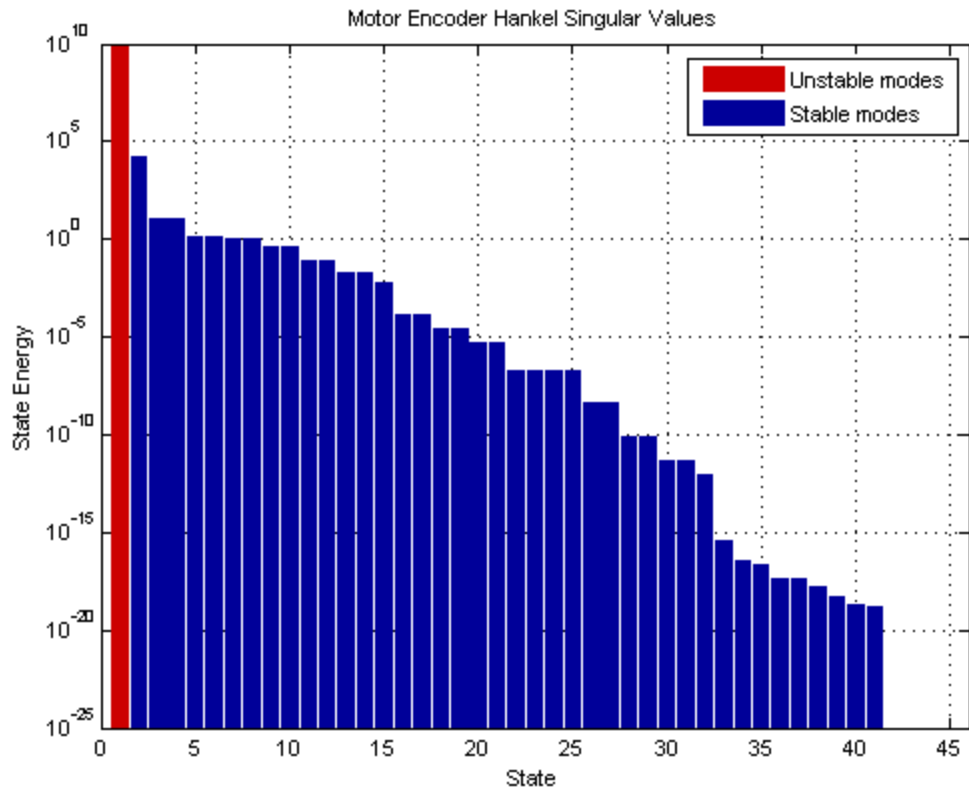
The full-state mechanical Simulink model for the azimuth drive axis with individual gearbox member stiffness and other variables perturbed produces a large, complex state-space model that includes many modal contributions that weakly contribute to the overall response. The complete model for the motor and absolute encoder outputs for a common input signal is of order 45 for each output.

The model order reduction process includes the following steps:

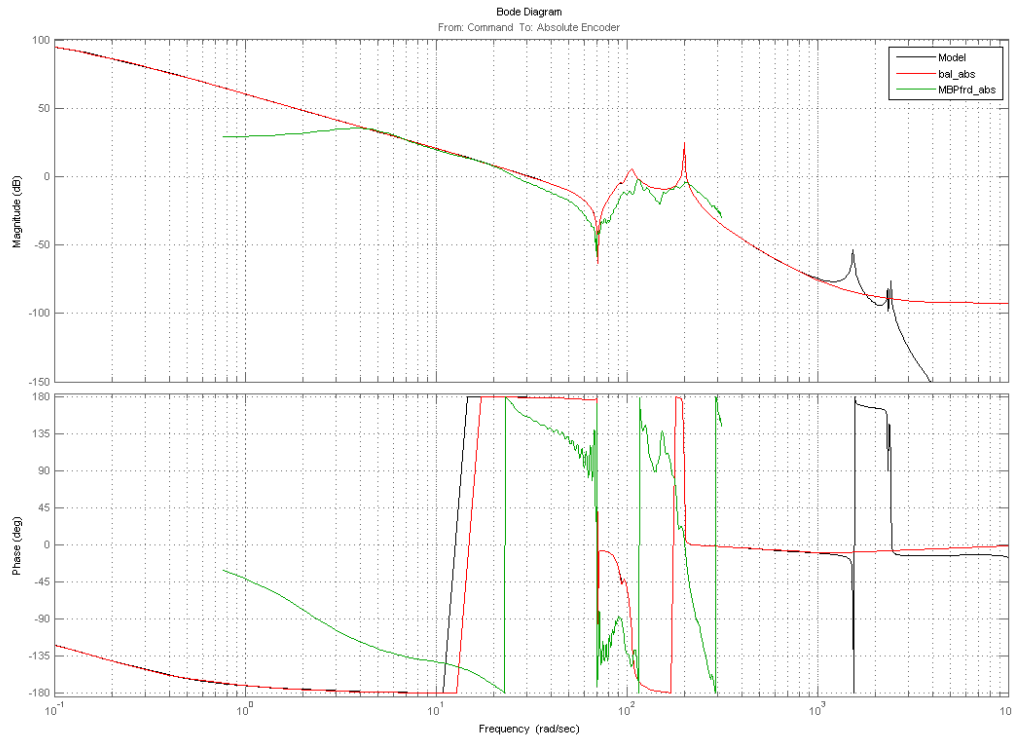
1. Compute the Hankel singular values (HSV) for each model.
2. Calculate balancing data to use in re-computation of the system matrices to lower the models' sensitivity to perturbations of matrix entries.
3. Use knowledge of the HSV and balancing data to select the model orders to discard.

Finally, the reduced-order balanced models are fit to the measured data. The Matlab Control Systems Toolbox makes available several scripts and other tools to facilitate the model identification and reduction process.

For HSV decomposition, the Matlab utility *hsvd* is used to process a given LTI (Linear Time-Invariant) model object. The model objects produced by the Simulink mechanical system model are partitioned by the *hsvd* algorithm into unstable and stable modes, and then the individual modes have their total energy contribution to the overall system response calculated. An example of an unstable mode is a double integrator, where the output grows without bound for any non-zero input. Since the mechanical model in fact contains such a double integrator, we expect at least one unstable mode from the HSV output. On the next page, the *hsvdplot* output for the absolute encoder and motor encoder models is shown.

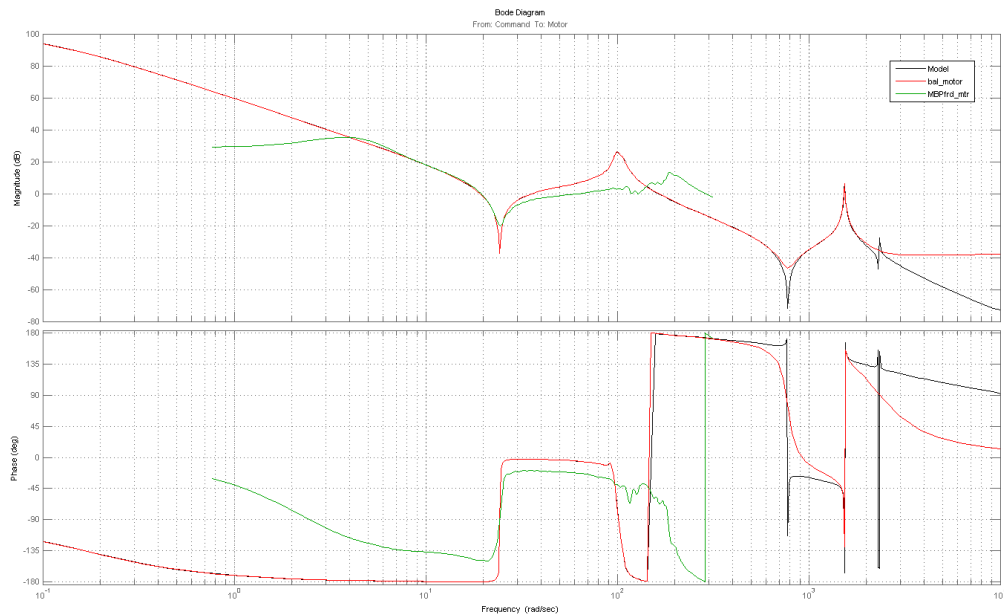


As can be seen, most of the energy is in the first 12 or so orders, and so the higher model orders can be discarded. Proof of this is shown below, where we overlay the measured data, the full-order model output, and the reduced-order model:



The reduced-order model and the full-order models naturally agree up to about 1000 rad/s. Their overall agreement with the measured data from manipulation of the mechanical model remains the same, as was reported earlier.

For the motor encoder model output, we have the same (next page):

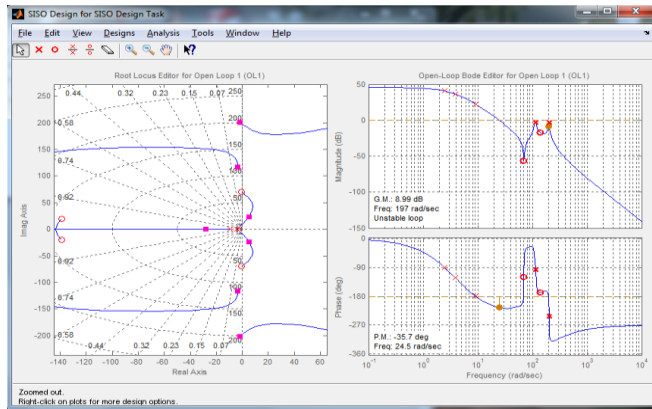
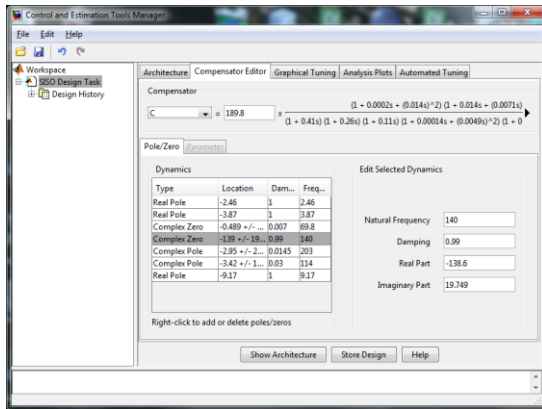


Again, we have good agreement on the balanced, reduced-order model out to about 1000 rad/s. Since the model are both LTI and each has a double integrator, the low-frequency response doesn't agree below about 5 rad/s.

Improving Model Fit

Once a balanced reduced-order model is in hand, the next part of improving the model fit is done via manual manipulation of the model pole and zero locations and their damping (for complex poles and zeros). Some work was done over the past few weeks to create a tool to directly overlay the target measured response with an adjustable response from the model to streamline this work but was abandoned due to time constraints. Instead of spending a lot of time creating a new tool, the built-in design GUI from the Control System Toolbox, *sisotool*, was used to tweak the models to improve the model fit.

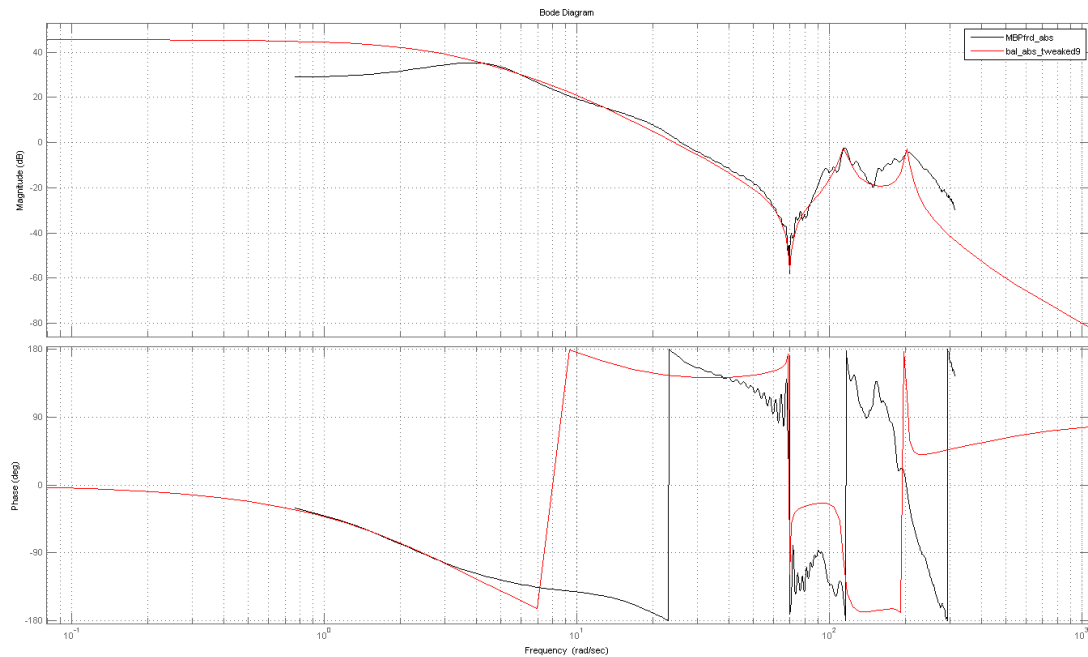
Sisotool is normally used to produce a compensator for a given single-input single-output (SISO) system; in this mode, the compensator is replaced by the model to be adjusted. In the GUI, poles and zeros can be added, removed, and edited. The system's DC gain can be adjusted as well. For this, the DC gains of both the motor and absolute encoders were adjusted to the same value since we normally set the encoder output scaling to the same (telescope arcseconds).

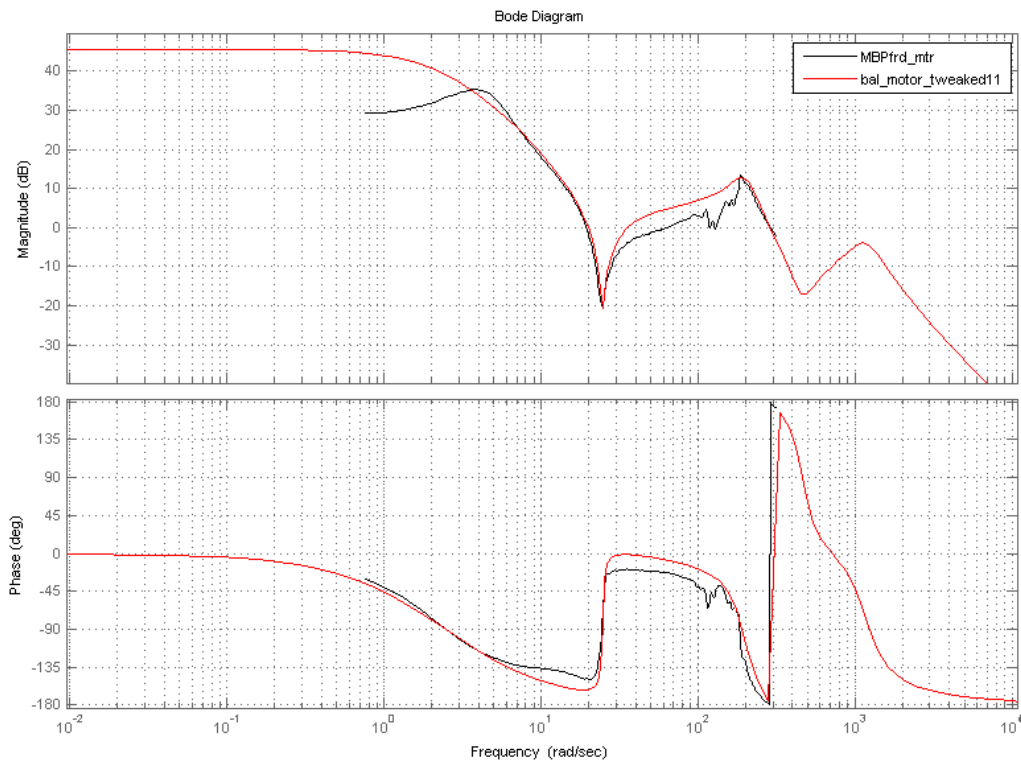


Sisotool GUI

The models to be adjusted are inserted into the GUI as a compensator object, and then the values in the left pane can be manually edited, or mouse click and drag inputs can operate on the open-loop graph locations.

Iterating on the open-loop response, and comparing to the measured data, we arrive at the following two Bode plot overlays:





Conclusion

The model orders have been reduced to 7 for the absolute encoder model and 6 for the motor encoder model, and the overall fit for both has been improved. The fit still does not completely agree at low frequencies mainly due to the models being strictly linear and not inclusive of friction, which is the major contributor at those low frequencies.

The models shown here can be used for frequency-response based design studies. The full-state mechanical model is good enough for use in time simulation of the system behavior – primarily friction and gear-noise modeling. The next report in this series will describe the results of initial linear controller design and time simulation studies.